The MIT Press Series in Artificial Intelligence
Edited by Patrick Henry Winston and Michael Brady

Artificial Intelligence: An MIT Perspective, Volume I: Expert Problem Solving, Natural Language Understanding, Intelligent Computer Coaches, Representation and Learning edited by Patrick Henry Winston and Richard Henry Brown, 1979

Artificial Intelligence: An MIT Perspective, Volume II: Understanding Vision, Manipulation, Computer Design, Symbol Manipulation edited by Patrick Henry Winston and Richard Henry Brown, 1979

NETL: A System for Representing and Using Real-World Knowledge by Scott Fahlman, 1979

The Interpretation of Visual Motion by Shimon Ullman, 1979

A Theory of Syntactic Recognition for Natural Language by Mitchell P. Marcus, 1980

Turtle Geometry: The Computer as a Medium for Exploring Mathematics by Harold Abelson and Andrea diSessa, 1981

From Images to Surfaces: A Computational Study of the Human Early Visual System by William Eric Leifur Grimson, 1981

Robot Manipulators: Mathematics, Programming and Control by Richard P. Paul, 1981

Computational Models of Discourse edited by Michael Brady and Robert C. Berwick, 1982

Robot Motion: Planning and Control by Michael Brady, John M. Hollerbach, Timothy Johnson, Tomas Lozano-Pèrez, and Matthew T. Mason, 1982

In-Depth Understanding: A Computer Model of Integrated Processing for Narrative Comprehension by Michael G. Dyer, 1983

Robotics Research: The First International Symposium edited by Michael Brady and Richard Paul, 1984

Robotics Research: The Second International Symposium edited by Hideo Hanafusa and Hirochika Inoue, 1985

Robot Hands and the Mechanics of Manipulation by Matthew T. Mason and J. Kenneth Salisbury, Jr., 1985

The Acquisition of Syntactic Knowledge by Robert C. Berwick, 1985

The Connection Machine by W. Daniel Hillis, 1985

Legged Robots that Balance by Marc H. Raibert, 1986

R2C1

Legged Robots That Balance

Marc H. Raibert

(1986)

The MIT Press

Cambridge, Massachusetts
London, England

Chapter 1

Introduction

This book is about machines that use legs to run. They are dynamic machines that balance themselves actively as they travel about the laboratory. The purpose of these machines is to learn about the principles of legged locomotion, particularly those underlying control and balance. Such principles can help us to understand animal locomotion and to build useful legged vehicles.

This first chapter explains why legged locomotion is an important problem, it provides the reader with some background on the general topic of legged machines, and it highlights the results reported in the chapters that follow.

Why Study Legged Machines?

Aside from the sheer thrill of creating machines that actually run, there are two serious reasons for exploring the use of legs for locomotion. One reason is mobility. There is a need for vehicles that can travel in difficult terrain, where existing vehicles cannot go. Wheels excel on prepared surfaces such as rails and roads, but most places have not yet been paved. Only about half the earth's landmass is accessible to existing wheeled and tracked vehicles, whereas a much larger fraction can be reached by animals on foot. It should be possible to build legged vehicles that can go to the places that animals are already able to reach.

One reason legs provide better mobility in rough terrain than do wheels or tracks is that they can use isolated footholds that optimize support and traction, whereas a wheel requires a continuous path of support. As a

Figure 1.1. Legged systems do not require a continuous path of support. They can use isolated footholds that are separated by unusable terrain.

consequence, the mobility of a legged system is generally limited by the best footholds in the reachable terrain and a wheel is limited by the worst terrain. A ladder provides a good example—its steepest parts prohibit ascent on wheels, while the flattest parts, the rungs, enable ascent using legs (figure 1.1).

Another advantage of legs is that they provide an active suspension that decouples the path of the body from the paths of the feet. The payload is free to travel smoothly despite pronounced variations in the terrain. A legged system can also step over obstacles. In principle, the performance of legged vehicles can, to a great extent, be independent of the detailed

roughness of the ground. A legged system uses this decoupling to increase its speed and efficiency on rough terrain.

The construction of useful legged vehicles depends on progress in several areas of engineering and science. Legged vehicles will need systems that control joint motions, cycle the use of legs, monitor and manipulate balance, generate motions to use known footholds, sense the terrain to find good footholds, and calculate negotiable foothold sequences. Most of these tasks are not well understood yet, but research is under way. If this research is successful, it will lead to the development of legged vehicles that travel efficiently and quickly in terrain where softness, grade, or obstacles make existing vehicles ineffective. Such vehicles will be useful in industrial, agricultural, and military applications.

A second reason for exploring machines that use legs for locomotion is to understand human and animal locomotion. One need watch only a few instant replays on television to be awed by the variety and complexity of ways athletes can carry, swing, toss, glide, and otherwise propel their bodies through space, maintaining orientation, balance, and speed as they go. Such performance is not limited to professional athletes; behavior at the local playground is equally impressive from a mechanical engineering, sensory-motor integration, or computational point of view. Perhaps most exciting is the sight of one's own child advancing rapidly from creeping and crawling to walking, running, hopping, jumping, and climbing.

Animals also demonstrate great mobility and agility. They move quickly and reliably through forest, swamp, marsh, and jungle, and from tree to tree. Sometimes they move with great speed, often with great efficiency.

Despite excellence in using our own legs for locomotion, we are still at a primitive stage in understanding the control principles that underlie walking and running. What control mechanisms do animals use? One way to learn more about plausible mechanisms for animal locomotion is to build machines that locomote using legs. To the extent that an animal and a machine perform similar locomotion tasks, their control systems and mechanical structures must solve similar problems. By building machines, we can get new insights into these problems, and we can learn about possible solutions. Of particular value is the rigor required to build physical machines that actually work. Concrete theories and algorithms can guide biological research by suggesting specific models for experimental testing and verification. This sort of interdisciplinary approach is already becoming popular in other areas where biology and robotics have a common ground, such as vision, speech, and manipulation.

Dynamics and Balance Improve Mobility

The work reported in this book focuses on a dynamic treatment of legged locomotion, with particular attention to balance. This means that the legged systems studied operate in a regime where the velocities and kinetic energies of the masses are important determinants of behavior. In order to predict and influence the behavior of a dynamic system, one must consider the energy stored in each mass and spring as well as the geometric structure and configuration of the mechanism. Geometry and configuration taken alone do not provide an adequate model when a system moves with substantial speed or has large mass. Consider, for example, a fast-moving vehicle that would tip over if it stopped suddenly with its center of mass too close to the front feet.

The exchange of energy among its various forms is also important in understanding the dynamics of legged locomotion. For example, there is a cycle of activity in running that changes the form of the stored energy several times: the body's potential energy of elevation changes to kinetic energy during falling, then to strain energy when parts of the leg deform elastically during rebound with the ground, then into kinetic energy again as the body accelerates upward, and finally back into potential energy of elevation. This sort of dynamic exchange is central to an understanding of legged locomotion.

A dynamic treatment, however, does not imply an intractable treatment. Although the detailed dynamics of a legged system may indeed be complicated, control techniques that use dynamics may be simple. For example, if hopping is primarily a resonant bouncing motion, then a control system with the task of regulating hopping need not actively servo the body along a specified trajectory. It can stimulate and modulate the bouncing motion by delivering a thrust of the right magnitude just once during each cycle. Control systems can generally be made simpler if they are attuned to the dynamics of the mechanism they control and to the task the mechanism performs. A specific goal of the work reported in this book is to identify and explore control techniques that use dynamics in simple ways.

Dynamics also plays a role in giving legged systems the ability to balance actively.[1] A statically balanced system avoids tipping and the ensuing horizontal accelerations by keeping the center of mass of the body over the polygon of support formed by the feet. The feet and body move according to gait patterns that maintain this support relationship, as shown in figure 1.2. Animals sometimes use this sort of balance when they move slowly, but they usually balance actively.

[1] The terms "active balance," "dynamic balance," and "dynamic stability" are used interchangeably in this book. "Passive balance," "static balance," and "static stability" are also used interchangeably.

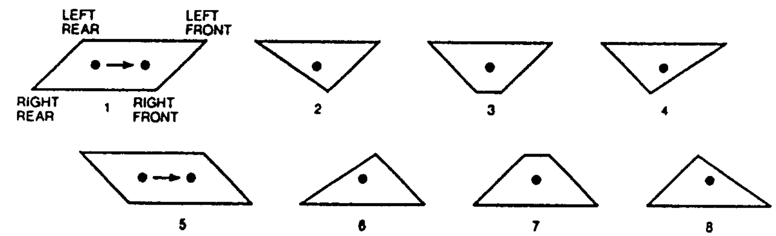
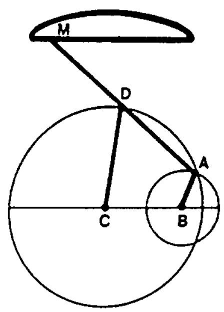


Figure 1.2. Statically stable gait. The diagram shows the sequence of support patterns provided by the feet of a crawling quadruped. The body and legs move to keep the projection of the center of mass within the polygon defined by the feet. A supporting foot is located at each vertex. The dot indicates the projection of the center of mass. Adapted from McGhee and Frank (1968).

A legged system that balances actively can tolerate departures from static equilibrium. Unlike a statically balanced system, which must always operate in or near equilibrium, an actively balanced system is permitted to tip and accelerate for short periods of time. The control system manipulates body and leg motions to ensure that each tipping interval is brief and that each tipping motion in one direction is compensated by a tipping motion in the opposite direction. An effective base of support is maintained over time. A system that balances actively may also permit vertical acceleration, such as the bouncing that occurs when the legs deform elastically and the ballistic travel that occurs between bounces.

The ability of an actively balanced system to depart from static equilibrium relaxes the rules on how legs can be used for support. This leads to improved mobility. For example, if a legged system can tolerate tipping, then it can position the feet far from the center of mass in order to use footholds that are widely separated or erratically placed. If it can remain upright with a small base of support, then it can travel where obstructions are closely spaced or where the path of firm support is narrow. The ability to tolerate intermittent support also contributes to mobility. It allows a system to move all its legs to new footholds at one time, to jump onto or over obstacles, and to use ballistic motions for increased speed. These abilities to use narrow base and intermittent support generally increase the types of terrain a legged system can negotiate. Animals routinely exploit active balance to travel quickly on difficult terrain. Legged vehicles will have to balance actively, too, if they are to move with animal-like mobility and speed.



$$CD = AD = DM = \frac{3+\sqrt{7}}{2}$$

$$BD = \frac{4+\sqrt{7}}{3}$$

Figure 1.3. Linkage used in an early walking machine. When the input crank AB rotates, the output point M moves along a straight path during part of the cycle and an arched path during the other part of the cycle. Two identical linkages are arranged to operate out of phase so at least one provides a straight motion at all times. The body is always supported by the feet connected to the straight-moving linkage. After Lucas (1894).

Research on Legged Machines

Before introducing the main topic of this book, the study of machines that run using active balance, we turn briefly to an account of previous work on legged machines.

The scientific study of legged locomotion began just over a century ago when Leland Stanford, then Governor of California, commissioned Eadweard Muybridge to find out whether or not a trotting horse left the ground with all four feet at the same time. Stanford had wagered that it never did. After Muybridge proved him wrong with a set of stop-motion photographs that would appear in *Scientific American* in 1878, Muybridge went on to document the walking and running behavior of over forty mammals, including humans (Muybridge 1955, 1957). His photographic data are still of considerable value and survive as a landmark in locomotion research.

The study of walking machines also had its origin in Muybridge's time. An early walking model appeared in about 1870. It used a kinematic linkage (figure 1.3) to move the body along a straight horizontal path while the

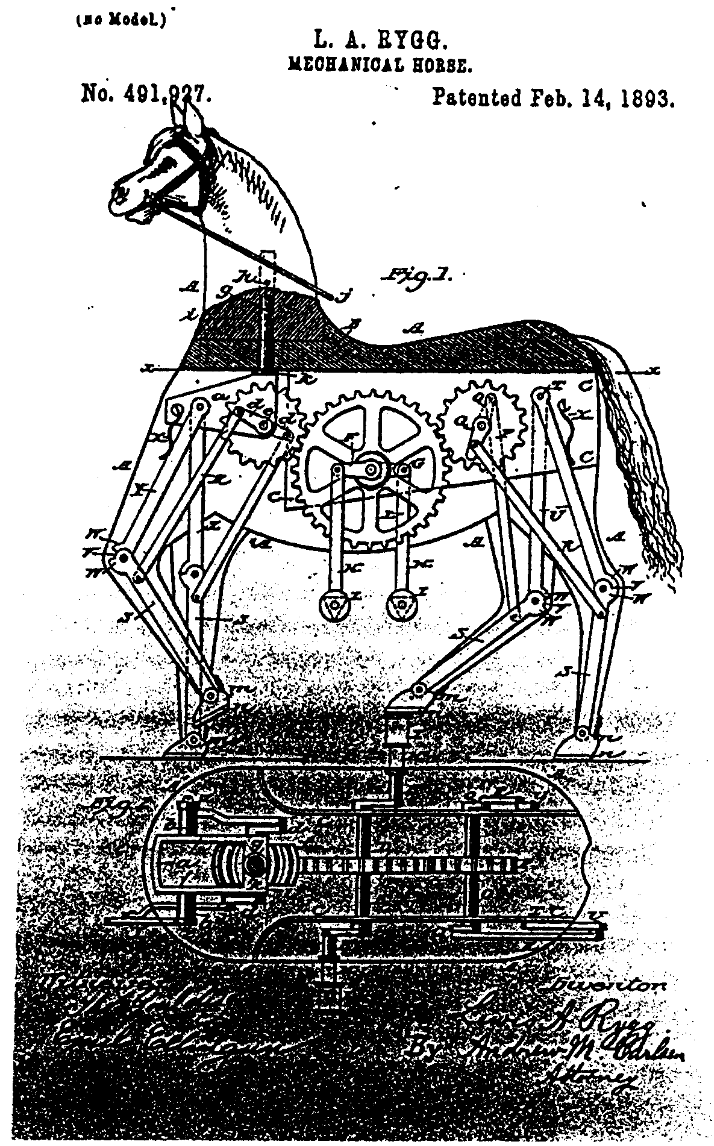


Figure 1.4. Mechanical horse patented by Lewis A. Rygg in 1893. The stirrups double as pedals so the rider can power the stepping motions. The reins move the head and forelegs from side to side for steering. Apparently the machine was never built.

feet moved up and down to exchange support during stepping. The linkage was originally designed by the famous Russian mathematician Chebyshev some years earlier (Lucas 1894). During the eighty or ninety years that followed, workers viewed the task of building walking machines as the task of designing kinematic linkages that would generate suitable stepping motions when driven by a source of power. Many designs were proposed (Rygg 1893, Nilson 1926, Ehrlich 1928, Kinch 1928, Snell 1947, Urschel 1949, Shigley 1957, Corson 1958, Bair 1959, Morrison 1968), but the performance of such machines was limited by their fixed patterns of motion which could not adjust to variations in the terrain. By the late 1950s it had become clear that a linkage providing fixed motion would not do the trick and that useful walking machines would need *control* (Liston 1970).

One approach to control was to harness a human. Ralph Mosher used this approach in building a four-legged walking truck at General Electric in the mid-1960s (Liston and Mosher 1968). The project was part of a decade-long campaign to build better teleoperators, capable of providing better dexterity through high-fidelity force feedback. The machine Mosher built stood 11 ft tall, weighed 3000 lbs, and was powered hydraulically. It is shown in figure 1.5. Each of the driver's limbs was connected to a handle or pedal that controlled one of the truck's four legs. Whenever the driver caused a truck leg to push on an obstacle, force feedback let the driver feel the obstacle as though it were his or her own arm or leg doing the pushing.

After about twenty hours of training Mosher was able to handle the machine with surprising agility. Films of the machine operating under his control show it ambling along at about 5 mph, climbing a stack of railroad ties, pushing a foundered jeep out of the mud, and maneuvering a large drum onto some hooks. Despite its dependence on a well-trained human for control, this walking machine was a landmark in legged technology, and it continues to be a significant advance over many of its successors.

An alternative to human control of legged vehicles became feasible in the 1970s: the use of a digital computer. Robert McGhee's group at Ohio State University was the first to use this approach successfully in 1977 (McGhee 1983). They built an insectlike hexapod that could walk with a number of standard gaits, turn, crab, and negotiate simple obstacles. The computer's primary task was to solve kinematic equations in order to coordinate the eighteen electric motors driving the legs. This coordination ensured that the machine's center of mass stayed over the polygon of support provided by the feet while allowing the legs to cycle through a gait. The machine traveled quite slowly, covering several meters per minute.

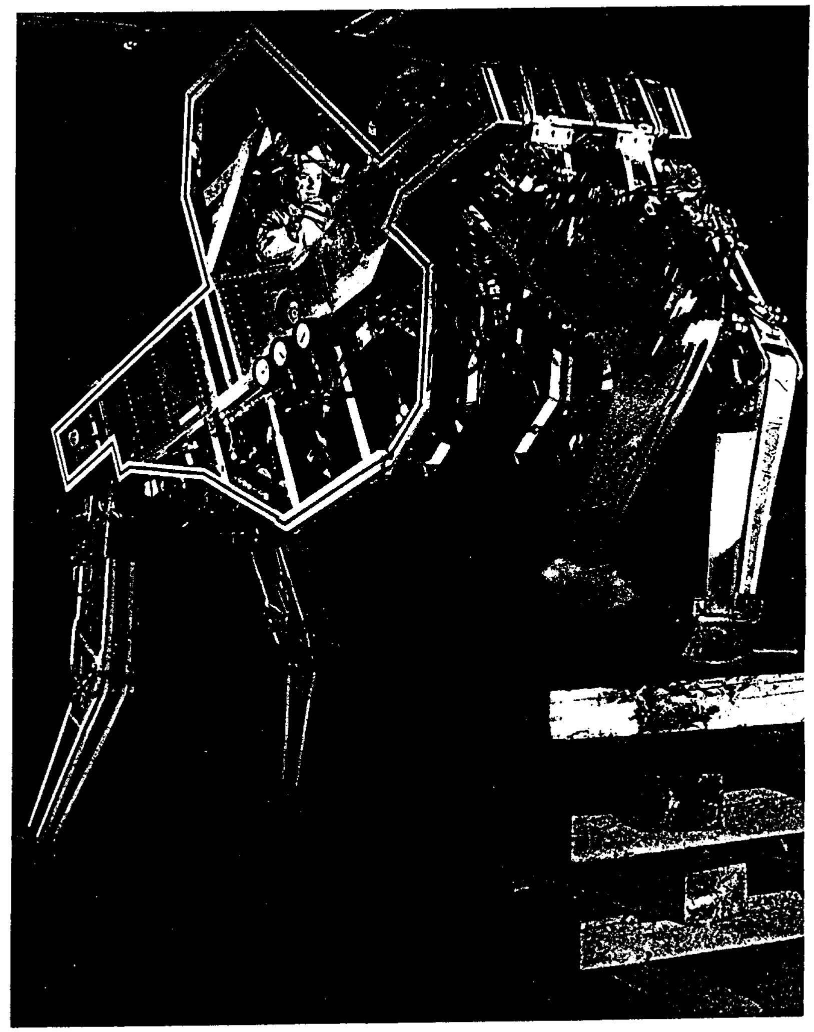


Figure 1.5. Walking truck developed by Ralph Mosher at General Electric in about 1968. The human driver controlled the machine with four handles and pedals that were connected to the four legs hydraulically. Photograph courtesy of General Electric Research and Development Center.

Force and visual sensing provided a measure of terrain accommodation in later developments (McGhee 1980, Klein and Briggs 1980, Ozguner et al. 1984). The hexapod provided McGhee with an excellent opportunity to pursue his earlier theoretical findings on the combinatorics and selection of gait (McGhee 1968, McGhee and Jain 1972, Koozekanani and McGhee 1973). The group at Ohio State is currently building a much larger hexapod, about 3 tons, that is intended to operate on rough terrain with a high degree of autonomy (Waldron et al. 1984).

Gurfinkel and his co-workers in the USSR built a machine with characteristics and performance quite similar to McGhee's at about the same time (Gurfinkel et al. 1981). It used a hybrid computer for control, with heavy use of analog computation for low-level functions.

Hirose realized that linkage design and computer control were not mutually exclusive. His experience designing clever and unusual mechanisms—he had built seven kinds of mechanical snake—led to a special leg that simplified the control of locomotion and could improve efficiency (Hirose and Umetani 1980, Hirose et al. 1984). The leg was a three-dimensional pantograph that translated the motion of each actuator into a pure Cartesian translation of the foot. With the ability to generate x, y, and z translations of each foot by merely choosing an actuator, the control computer was freed from the arduous task of performing kinematic solutions. Actually, the mechanical linkage was helping to perform the calculations needed for locomotion. The linkage was efficient because the actuators performed only positive work in moving the body forward.

Hirose used this leg design to build a small quadruped, about 1 m long. It was equipped with touch sensors on each foot and an oil-damped pendulum attached to the body. Simple algorithms used the sensors to control actions of the feet. For instance, if a touch sensor indicated contact while the foot was moving forward, the leg would move backward a little bit, move upward a little bit, then resume its forward motion. If the foot had not cleared the obstacle, the cycle would repeat. The use of several simple algorithms like this one permitted Hirose's machine to climb up and down stairs and to negotiate other obstacles without human intervention (Hirose 1984).

These three walking machines, McGhee's, Gurfinkel's, and Hirose's, represent a class called *static crawlers*. Each differs in the details of construction and in the computing technology used for control, but they share a common approach to balance and stability. Enough feet are kept on the ground to guarantee a broad base of support at all times, and the body and legs move to keep the center of mass over this broad support base.

The forward velocity is low enough to predict stability based on the spatial configuration of the body and feet, without worrying about stored energy. Each of these machines has been used to study rough terrain locomotion in the laboratory, including experiments on terrain sensing, gait selection, and selection of foothold sequences. Several other machines that fall into this class have been studied in the intervening years (e.g., Russel 1983, Sutherland and Ullner 1984, Ooka et al. 1985).

Research on Active Balance

The last section focused on legged machines that use static techniques for balance. We now turn to the study of dynamic machines that balance actively. The first machines that balanced actively were automatically controlled inverted pendulums. Everyone knows that a human can balance a broom on his finger with relative ease. Why not use automatic control to build a broom that can balance itself?

Claude Shannon was probably the first to do so. In 1951 he used the parts from an erector set to build a machine that balanced an inverted pendulum atop a small powered truck (Shannon 1985). The truck drove back and forth in response to the tipping movements of the pendulum, as sensed by a pair of switches at its base. In order to move from one place to another, the truck first had to drive away from the goal to unbalance the pendulum toward the goal. In order to balance again at the destination, the truck moved past the destination until the pendulum was again upright with no forward velocity. It then moved back to the goal.

It was at Shannon's urging that Cannon and two of his students at Stanford University set about demonstrating controllers that balanced two pendulums at once. In one case the pendulums were mounted side by side on the cart, and in the other case they were mounted one on top of the other (figure 1.6). Cannon's group was interested in the single-input multiple-output problem and in the limitations of achievable balance: how could they use the single force that drove the cart's motion to control the angle of two pendulums as well as the position of the cart? How far from balance could the system deviate before it was impossible to return to equilibrium, given the parameters of the mechanical system, e.g., cart motor strength or pendulum lengths.

Using analysis based on normal coordinates and bang-bang switching curves, they expressed regions of controllability as explicit functions of the physical parameters of the system. Once these regions were found, their

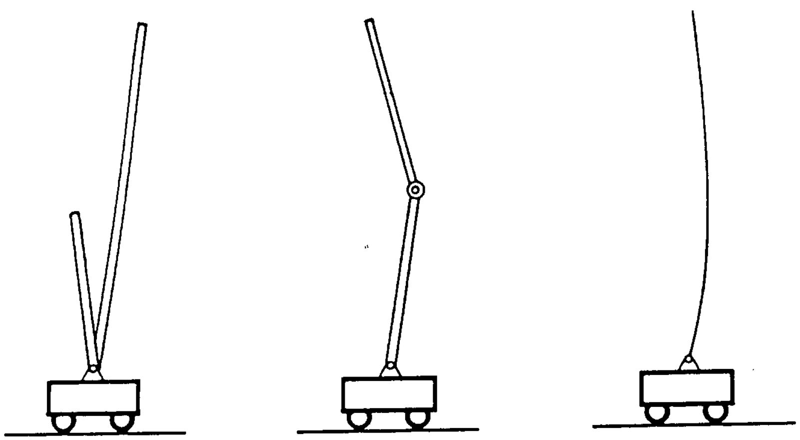


Figure 1.6. Cannon and his students built machines that balanced inverted pendulums on a moving cart. They balanced two pendulums side by side, one pendulum on top of another, and a long limber inverted pendulum. Only one input, the force driving the cart horizontally, was available for control. Adapted from Schaefer and Cannon (1966).

boundaries were used to find switching functions that provided control (Higdon and Cannon 1963, Higdon 1963). Later, they extended these techniques to provide balance for a flexible inverted pendulum (Schaefer 1965, Schaefer and Cannon 1966). These studies of balance for inverted pendulums were important precursors to later work on locomotion. The inverted pendulum model for walking would become the primary tool for studying balance in legged systems (e.g., Hemami and Weimer 1974, Vukobratovic and Stepaneko 1972, Vukobratovic 1973, Hemami and Golliday 1977, Kato et al. 1983, Miura and Shimoyama 1984). It is unfortunate that no one has yet extended Cannon's elegant analytical results to the more complicated legged case.

Mosher's group at General Electric was also interested in balance. Their original intention, before deciding to build the quadruped described earlier, was to build a walking biped that would be controlled by a human who would "walk" in an instrumented harness inside the cockpit. They started with a human factors experiment because they were unsure of the human's ability to adjust to the exaggerated vestibular input that would be experienced when one drives a machine several times taller than one's self. In the experiments the subjects stood on an inverted pendulum about 20 ft tall. The pendulum had pivots like an ankle and hip, one at the

floor and one just below the platform that supported the subject. These pivots were servoed to follow the corresponding ankle and hip motions of the subject. All eighty-six people tested learned to balance the machine in less than fifteen minutes, and most learned in just two or three (Liston and Mosher 1968). Although the GE walking truck mentioned earlier could, in principle, operate using purely static techniques, the driver's ability to balance it actively probably contributed to its smooth operation. A GE walking biped was never built.

The importance of active balance in legged locomotion had been widely recognized for some years (e.g., Manter 1938, McGhee and Kuhner 1969, Frank 1970, Vukobratovic 1973, Gubina, Hemami, and McGhee 1974, Beletskii 1975a), but progress in building physical legged systems that employ such principles was retarded by the perceived difficulty of the task. It was not until the late 1970s that experimental work on balance in legged systems got underway.

Kato and his co-workers built a biped that walked with a *quasi-dynamic gait* (Ogo et al. 1980, Kato et al. 1983). The machine had ten hydraulically powered degrees of freedom and two large feet. Generally, this machine was a static crawler, moving along a preplanned trajectory to keep the center of mass over the base of support provided by the supporting foot. Once during each step, however, the machine temporarily destabilized itself to tip forward so that support would be transferred quickly from one foot to the other. Before the transfer took place on each step, the *catching foot* was positioned so that it would return the machine to equilibrium passively, without requiring an active response. A modified inverted pendulum model was used to plan the tipping motion.

In 1984 the machine walked with a quasi-dynamic gait, taking about a dozen 0.5 m long steps per minute (Takanishi et al. 1985). The use of a dynamic transfer phase makes an important point: A legged system can employ complicated dynamic behavior without requiring a very complicated control system.

Miura and Shimoyama (1980, 1984) built the first walking machine that really balanced actively. Their *stilt biped* was patterned after a human walking on stilts. Each foot provided only a point of support, and the machine had three actuators: one for each leg that moved the leg sideways and a third that separated the legs fore and aft. Because the legs did not change length, the hips were used to pick up the feet. This gave the machine a pronounced shuffling gait like Charlie Chaplin's stiff-kneed walk.

Once again, control for the stilt biped relied on an inverted pendulum model of its behavior. Each time a foot was placed on the floor, its position was chosen according to the tipping behavior that was expected from an inverted pendulum. Actually, the problem was broken down as though there were two pendulums, one in the pitching plane and one in the rolling plane. The choice of foot position along each axis took the current and desired state of the system into account. In order to perform the necessary calculations, the control system used tabulated descriptions of planned leg motions together with linear feedback. Unlike Kato's machine, which came to static equilibrium before and after each dynamic transfer, the stilt biped tipped all the time.

Matsuoka was the first to build a machine that ran, where running is defined by periods of ballistic flight with all feet leaving the ground. His goal was to model repetitive hopping in the human. He formulated a model consisting of a body and one massless leg and he simplified the problem by assuming that the duration of the support phase was short compared with the ballistic flight phase. This extreme form of running, for which nearly the entire cycle was spent in flight, minimized the influence of tipping during support. This model permitted Matsuoka to derive a time-optimal state feedback controller that provided stability for hopping in place and for low speed translations (Matsuoka 1979).

To test his method for control, Matsuoka built a planar one-legged hopping machine. The machine operated at low gravity by lying on a table inclined 10° from the horizontal, rolling on ball bearings. An electric solenoid provided a rapid thrust at the foot, so the support period was short. The machine hopped in place at about 1 hop/s and traveled back and forth on the table.

Introduction to Running Machines

Running is a special form of legged locomotion that uses ballistic flight phases to obtain high speed. To study running, my co-workers and I have explored a variety of legged systems and implemented some of them in the form of physical machines. In the course of this work we have identified a number of simple ideas about the control of legged locomotion, and have applied them to demonstrate machines that run and balance. The purpose of this work is to provide a foundation of knowledge that can lead to both the construction of useful legged vehicles and to a better understanding of legged locomotion as it occurs in nature. This section is an overview of this work on running and a summary of the main findings; details are found in the chapters that follow.

Table 1.1. Milestones in legged technology.

When	Who	What
1850	Chebyshev	Designs linkage used in early walking mechanism (Lucas 1894).
1872	Muybridge	Uses stop-motion photography to document running animals.
1893	Rygg	Patents human-powered mechanical horse.
1945	Wallace	Patents hopping tank. Reaction wheels provide stability.
1961	Space General	Eight-legged kinematic machine walks in outdoor terrain (Morrison 1968).
1963	Cannon, Higdon & Schaefer	Control system balances single, double, and limber inverted pendulums.
1968	Frank & McGhee	Simple digital logic controls walking of Phony Pony.
1968	Mosher	GE quadruped truck climbs railroad ties under control of human driver.
1969	Bucyrus-Erie Co.	Big Muskie, a 15,000 ton walking dragline is used for strip mining. It moves in soft terrain at 900 ft/hr (Sitek 1976).
1977	McGhee	Digital computer coordinates leg motions of hexapod walking machine.
1977	Gurfinkel	Hybrid computer controls hexapod walker in USSR.
1977	McMahon & Greene	Human runners set new speed records on *tuned track* at Harvard. Its compliance was adjusted to mechanics of human leg.
1980	Hirose & Umetani	Quadruped machine climbs stairs and over obstacles using simple sensors. The leg mechanism simplifies control.
1980	Kato	Hydraulic biped walks with quasi-dynamic gait.
1980	Matsuoka	Mechanism balances in the plane while hopping on one leg.
1981	Miura & Shimoyama	Walking biped balances actively in three dimensional space.
1983	Sutherland	Hexapod carries human rider. Computer, hydraulics, and human share computing task.
1983	Odetics	Self-contained hexapod lifts and moves back end of pickup truck (Russell 1983).

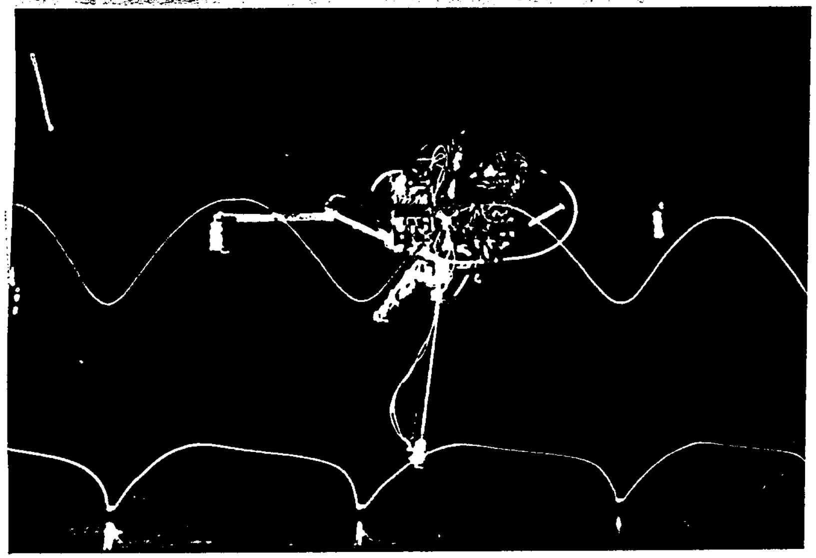


Figure 1.7. Planar hopping machine traveling at about 0.8 m/s (1.75 mph) from right to left. Lines made by light sources attached to the machine indicate paths of the foot and the hip.

It was to study running in its simplest form that we built a running machine that had just one leg. It ran by hopping like a kangaroo, using a series of leaps. A machine with only one leg draws attention to active balance and dynamics while postponing the difficult problems of coordinating the behavior of many legs. Active balance and dynamics are central issues for a one-legged machine, while gait and interleg coordination are of little concern. Gait has dominated thinking about legged locomotion for some years, and one wonders how central it really is. Are there algorithms for walking and running that are independent of gait or that work correctly for any number of legs? Perhaps a machine with just one gait could suggest answers to these questions.

The machine we built to study these problems had two main parts: a body and a leg. The body provided the main structure that carried the actuators and instrumentation needed for the machine's operation. The leg could telescope to change length and was springy along the telescoping axis. Sensors measured the pitch angle of the body, the angle of the hip, the length of the leg, the tension in the leg spring, and contact with the ground. This first machine was constrained to operate in a plane, so it could move only up and down and fore and aft and rotate in the plane. An umbilical cable connected the machine to power and a control computer.

For this machine running and hopping are the same. The running cycle has two phases. During one phase, called *stance* or *support*, the leg supports the weight of the body and the foot stays in a fixed location on the ground. During stance, the system tips like an inverted pendulum. During the other phase, called *flight*, the center of mass moves ballistically, with the leg unloaded and free to move.

Control of Running Was Decomposed into Three Parts

We were surprised to find that a simple set of algorithms could provide control for this planar one-legged hopping machine. The approach was to consider separately the hopping motion, forward travel, and posture of the body. This decomposition lead to a control system with three parts:

- *Hopping.* One part of the control system excited the cyclic hopping motion that underlies running while regulating how high the machine hopped. The hopping motion is an oscillation governed by the mass of the body, the springiness of the leg, and gravity. During support, the body bounced on the springy leg, and during flight, the system traveled a ballistic trajectory. The control system delivered a vertical thrust with the leg during each support period to sustain the oscillation and to regulate its amplitude. Some of the energy needed for each hop was recovered by the leg spring from the previous hop.
- *Forward Speed.* A second part of the control system for the one-legged hopping machine regulated the forward running speed and acceleration. This was done by moving the leg to an appropriate forward position with respect to the body during the flight portion of each cycle. The position of the foot with respect to the body when landing has a strong influence on the behavior during the support period that follows. Depending on where the control system places the foot, the body will continue to travel with the same forward speed, it will accelerate to go faster, or it will slow down. To calculate a suitable forward position for the foot, the control system takes account of the actual forward speed, the desired speed, and a simple model of the legged system's dynamics. A single algorithm works correctly when the machine is hopping in place, accelerating to a run, running at a constant speed, and slowing to a stationary hop.

- *Posture.* The third part of the control system stabilizes the pitch angle of the body to keep the body upright. Torques exerted between the body and leg about the hip accelerate the body about its pitch axis, provided that there is good traction between the foot and the ground. During the support period there is traction because the leg supports the load of the body. A linear servo operates on the hip actuator during each support period to restore the body to an upright posture.

An important simplification came from breaking running down into the control of the up and down bouncing motion, the forward speed, and body posture. Partitioning the control into these three parts made running much easier to understand and led to a fairly simple control system. The algorithms implemented to perform each part of the control task were themselves simple, although none was optimized for performance. The details of the individual control algorithms are not so important as the framework provided by the decomposition.

Using the three-part control system, the planar one-legged machine hopped in place, traveled at a specified rate, maintained balance when disturbed, and jumped over small obstacles. Top running speed was about 1.2 m/s (2.6 mph). The utility of the decomposition and framework was not limited to planar hopping on one leg—the approach was generalized for controlling a three-dimensional one-legged machine, a planar two-legged machine, and a quadruped.

Locomotion in Three Dimensions

The machine just described was constrained mechanically to operate in the plane, but useful legged systems must balance themselves in three-dimensional space. Can the control algorithms used for hopping in the plane be generalized somehow for hopping in three dimensions? A key to answering this question was the recognition that animal locomotion is primarily a planar activity, even though animals are three-dimensional systems. Films of a kangaroo hopping on a treadmill first suggested this point. One observes the legs sweeping fore and aft through large angles, the tail sweeping in counteroscillation with the legs, and the body bouncing up and down. These motions all occur in the sagittal plane, with little or no motion normal to the plane.

Sesh Murthy realized that the plane in which all this activity occurs can generally be defined by the forward velocity vector and the gravity vector. He called this the *plane of motion* (Murthy 1983). For a legged system without a preferred direction of travel, the plane of motion might

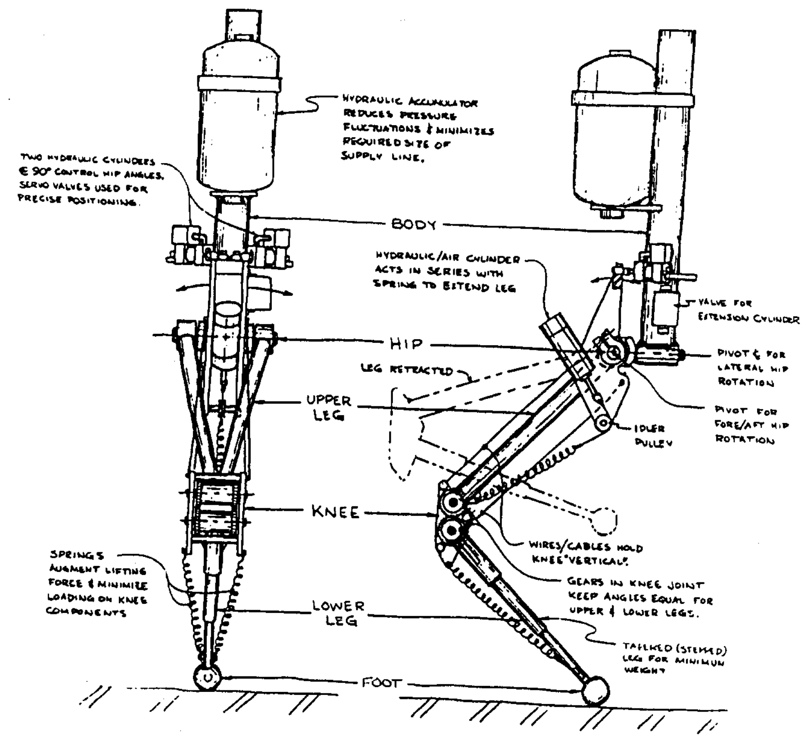


Figure 1.8. Ben Brown and I had this early concept for a one-legged hopping machine that was to operate in three dimensions. This version never left the drawing board.

vary from stride to stride, but it would be defined in the same way. We found that the three-part control system retained its effectiveness when used to control activity within the plane of motion.

We also found, however, that the mechanisms needed to control the remaining *extraplanar degrees of freedom* could be cast in a form that fit into the original three-part framework. For instance, the algorithm for placing the foot to control forward speed became a vector calculation. One component of foot placement determined forward speed in the plane of motion, whereas the other component caused the plane to rotate about a vertical axis, permitting the control system to steer. A similar extension applied to body posture. The result was a three-dimensional three-part control system that was derived from the one used for the planar case, with very little conceptual complication.

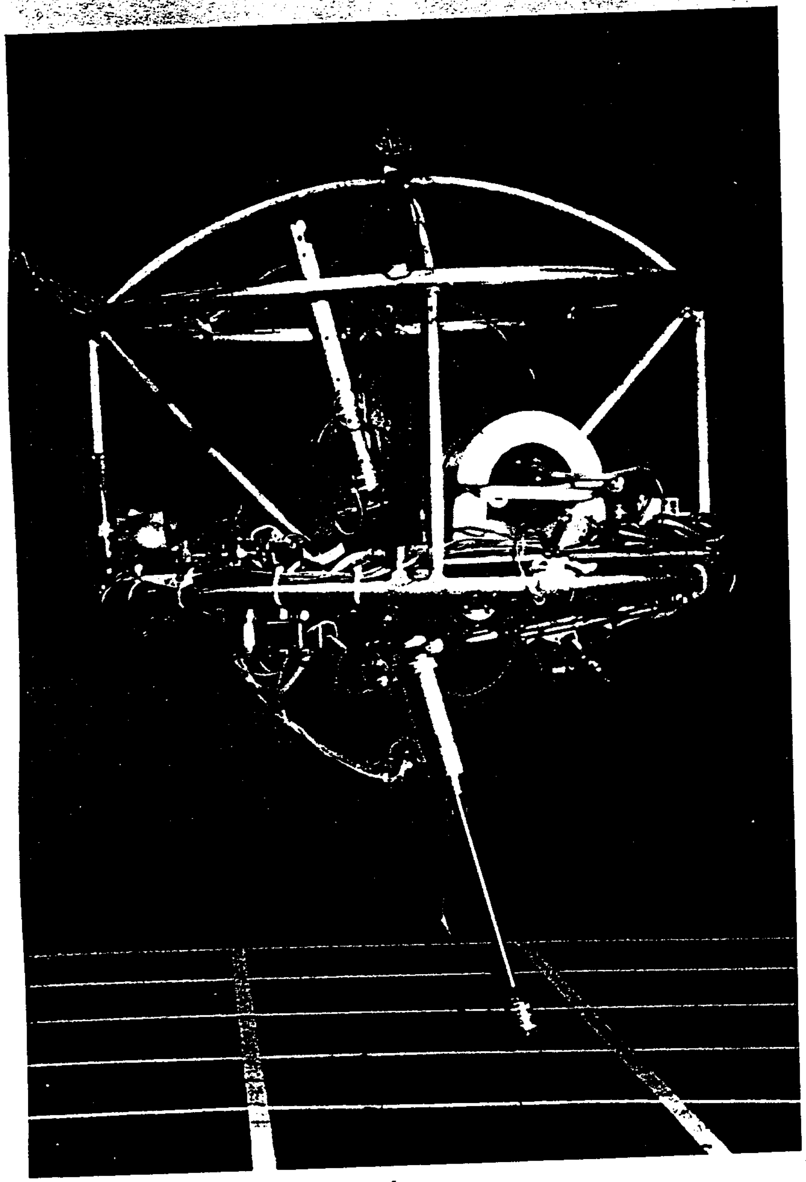


Figure 1.9. Three-dimensional hopping machine used for experiments. The control system operates to regulate hopping height, forward velocity, and body posture. Top recorded running speed was about 2.2 m/s (4.8 mph).

To explore these ideas, we built a second hopping machine, shown in figure 1.9. It had an additional joint at the hip to permit the leg to move sideways as well as fore and aft, and the machine had no external mechanical support. Otherwise, it was similar to the planar hopping machine described earlier. In operation this machine balanced itself as it hopped along simple paths in the laboratory, traveling with a top speed of 2.2 m/s (4.8 mph).

Running on Several Legs

Experiments on machines with one leg were not motivated by an interest in one-legged vehicles. Although such vehicles might very well turn out to have merit,[2] our interest was in getting at the basics of active balance and dynamics in the context of a simplified locomotion problem. In principle, results from machines with one leg could have value for understanding all sorts of legged systems, perhaps with any number of legs.

Given the successful control of machines that run and balance on one leg, can we use what we learned to understand and control machines with several legs? Our study of this problem has progressed in two steps. For a biped that runs like a human, with alternating periods of support and flight, the one-leg control algorithms apply directly. Because the legs are used in alternation, only one leg is active at a time: only one leg is placed on the ground at a time, only one leg thrusts on the ground at a time, and only one leg can exert a torque on the body at a time. We call this sort of running a *one-foot gait*. Assuming that the behavior of the other leg does not interfere, the one-leg algorithms for hopping, forward travel, and posture can each be used to control the active leg. Of course, to make this workable, some bookkeeping is required to keep track of which leg is active and to keep the extra leg out of the way.

Jessica Hodgins and Jeff Koechling demonstrated the effectiveness of this approach by using the one-leg algorithms to control each leg of a planar biped. The machine, shown in figure 1.10, has run at 4.3 m/s (9.5 mph). As one might guess, the biped can also travel by hopping on one leg, and it can switch back and forth between gaits. We found that it was very simple to extend the one-leg algorithms for two-legged running.

In principle, this approach could be used to control any number of legs, so long as just one is made active at a time. Unfortunately, when

[2] Wallace and Seifert saw merit in vehicles with one leg. Wallace (1942) patented a one-legged hopping tank that was supposed to be hard to hit because of its erratic movements. Seifert (1967) proposed the *Lunar Pogo* as a means of efficient travel on the moon.

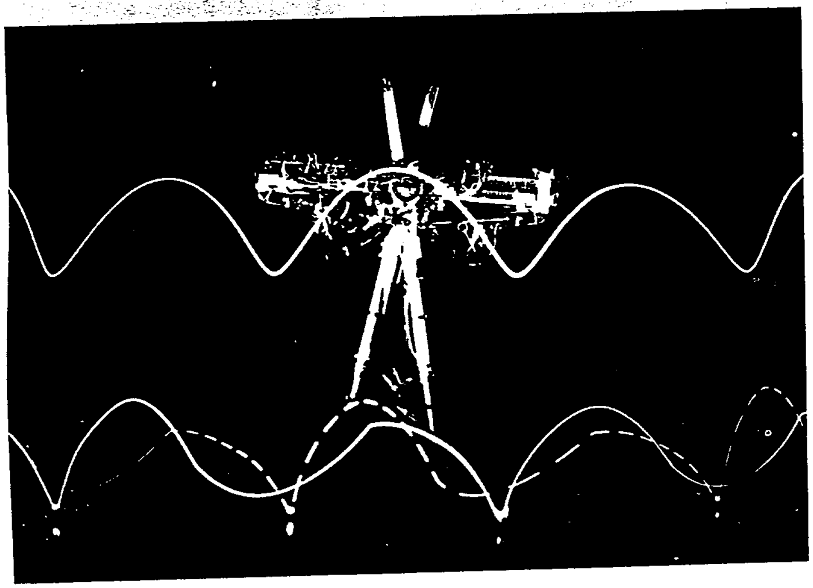


Figure 1.10. The planar biped can run with a gait that uses the legs in alternation like a human or with a hopping gait, and it can switch back and forth between gaits. The two legs counteroscillate during normal running. Top running speed was 4.3 m/s (9.5 mph). The control is based on the three-part decomposition originally used for the one-legged hopping machines. During one-legged hopping, the extra leg acts like a tail, swinging out of phase with the active leg. From Hodgins, Koechling and Raibert (1985).

there are several legs this is usually not feasible. Suppose, however, that a control mechanism coordinates legs that share support simultaneously, making them behave like a single equivalent leg—what Sutherland (1983) has called *a virtual leg*. Suppose further that more than one leg provides support at a time but that all support legs are coordinated to act like a virtual leg. One can then map several multi-legged gaits into *virtual biped one-foot gaits*. For example, the trotting quadruped maps into a virtual biped running with a one-foot gait.

We argue that the trotting quadruped is like a biped, that a biped is like a one-legged machine, and that control of one-legged machines is a solved problem. A control system for quadruped trotting could consist of a servo that coordinates each pair of legs to act like one virtual leg, a three-part control system that acts on the virtual legs, and a bookkeeping mechanism that keeps track. Figure 1.11 is a photograph of a four-legged machine that runs with precisely this sort of control system.

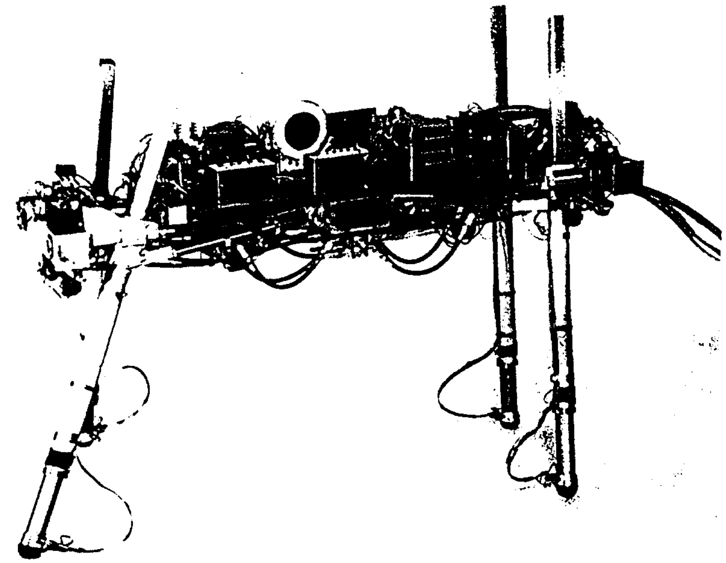


Figure 1.11. Quadruped machine that runs by trotting. *Virtual legs* are used to map trotting into biped running, which in turn is controlled with one-leg algorithms.

Symmetry in Robots and Animals

i.e. sum

In order to run at constant forward speed, the instantaneous forward accelerations that occur during a stride must integrate to zero. One way to satisfy this requirement is to organize running behavior so that forward acceleration has an odd symmetry throughout each stride—functions with odd symmetry integrate to zero over symmetric limits.[3] This sort of symmetry was used to control forward speed in all four machines just described. It was accomplished by choosing an appropriate forward position for the foot on each step. In principle, symmetry of this sort can be used to simplify locomotion in systems with any number of legs and for a wide range of gaits.

Can the symmetries developed for legged machines help us to understand the behavior of legged animals? To find out, we have examined film

[3] If $x(t)$ is an odd function of time, then $x(t) = -x(-t)$. If $x(t)$ is even, then $x(t) = x(-t)$.

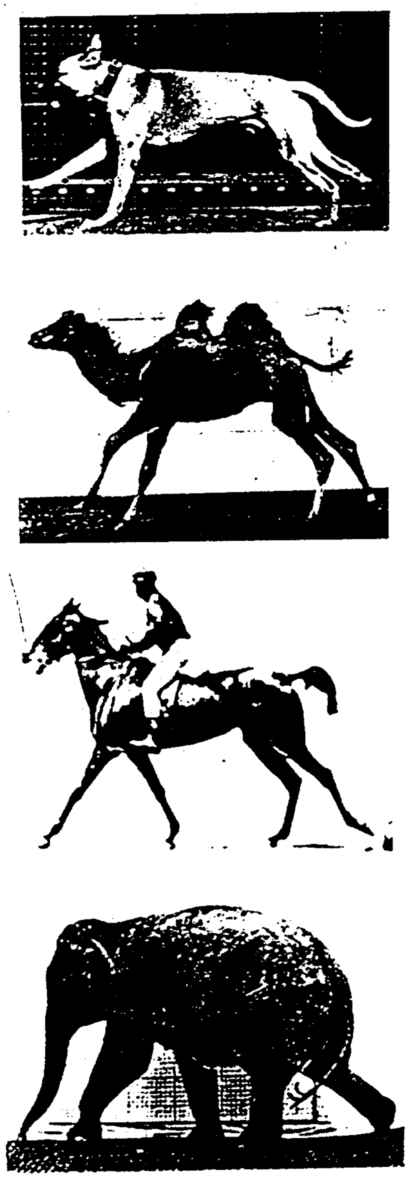


Figure 1.12. (Facing page.) Symmetry in animal locomotion. Animals shown in symmetric configuration halfway through the stance phase for several gaits: rotary gallop (top), transverse gallop (second), canter (third), and amble (bottom). In each case the body is at minimum altitude, the center of support is located below the center of mass, the rearmost leg was recently lifted, and the frontmost leg is about to be placed. Photographs from Muybridge (1957); reprinted with permission from Dover Press.

Table 1.2. Summary of progress in the CMU Leg Laboratory.

When	What
1982	Planar one-legged machine hops in place, travels at specified rate of up to 1.2 m/s (2.6 mph), tolerates mechanical disturbances, and jumps over small obstacles.
1983	One-legged hopping machine runs on open floor, balancing in three dimensions. Top speed about 2.2 m/s (4.5 mph).
1983	Murphy finds passively stabilized bounding gait for simulated quadruped-like model (Murphy 1984).
1984	Cat and human found to run with symmetry like running machines.
1984	Quadruped runs with trotting gait. *Virtual legs* permit use of one-leg control algorithms.
1985	Planar biped runs with one- and two-legged gaits and can change gait while running. Top speed is 4.3 m/s (9.5 mph). (Hodgins, Koechling and Raibert, 1985).

data for running animals and humans. In particular we have looked at a cat trotting and galloping on a treadmill and a human running on a track. The data conform reasonably well to the predicted even and odd symmetries. In some cases the data are remarkably symmetric.

Summary

Here is a brief summary of this introduction to running machines and of the following chapters (table 1.2):

- The goal of this work is to learn more about dynamics and active balance in legged locomotion, both for the purpose of building legged vehicles and to understand animal locomotion.
- A planar one-legged hopping machine can balance actively using separate control algorithms for hopping, forward speed, and posture.

- It is not much harder to provide control and balance for hopping in three dimensions than it is in two—the three-part control decomposition still applies.
- The one-leg control algorithms remain effective for biped running, requiring only some additional bookkeeping.
- Quadruped trotting can be implemented like biped running if the control system has a mechanism to coordinate pairs of legs.
- Symmetry is important for simplifying the control of legged robots and may be important in animal locomotion, too.

One caveat before ending this introductory discussion. Despite the goal of improved vehicular mobility in difficult terrain, legged vehicles have not yet proved themselves by moving out of the laboratory and into the bush. Several researchers are actively working toward this goal, but the research reported in this book avoids the issue of difficult terrain entirely. Although we take motivation from the need to travel on rough terrain, the running experiments reported here have not yet ventured beyond our very flat laboratory floor.

Additional Readings

To gain a broader background and learn more about problems, issues, and progress in legged locomotion, the following additional readings are recommended: Gabrielli and Von Karmen (1950) is the classic paper on the fundamental energetics of vehicular travel. Hirose (1984) follows up on the same theme. Bekker (1969) discusses the general problem of vehicular mobility in rough terrain in great detail. He includes a treatment of soil mechanics, an often neglected but important part of the story.

To learn more about current research on legged robots, see the special issue edited by Raibert (1984b). A companion video tape to the special issue available from the MIT Press shows several walking machines from Japan and the US in action. For a fascinating historical perspective on walking machines, see Liston (1970). To access the large body of theoretical work on legged machines, start with McGhee (1968), McGhee and Frank (1968), and Vukobratovic and Stepaneko (1972). Hemami and Golliday (1977) consider problems in control theory related to legged locomotion.

Margaria (1976) and McMahon (1984) introduce the biomechanics of animal locomotion, and Alexander and Goldspink (1977) and Hoyt and Taylor (1981) provide many interesting details. The story of how animals

use elastic storage in running is particularly relevant: Dawson and Taylor (1973), Cavagna, Heglund and Taylor (1977), McMahon and Greene (1978).

Hildebrand (1960) and Pearson (1976) provide general introductions to animal locomotion. For a collection that covers a wide range of locomotion topics in invertebrates and vertebrates, see Herman et al. (1976). Excellent reviews of research in the relevant neurophysiology are Grillner (1975) and Wetzel and Stuart (1976). Both reviews are brought up to date in the collection by Grillner et al. (1985). To learn more about how robotics and biology can interact productively, see the excellent discussion by Hildreth and Hollerbach (1985) and Marr (1976).

Chapter 2

Hopping on One Leg in the Plane

Running is like the bouncing of a ball (Margaria 1976). A ball falls under the acceleration of gravity until an elastic collision with the ground reverses its direction, sending it upward to be accelerated by gravity once again. During the collision, the ball first deforms to absorb its kinetic energy and then returns the kinetic energy as it recovers its original shape. The exchange dissipates a fraction of the energy. The overall bouncing pattern oscillates between ballistic flight phases and elastic collisions until the ball's energy is dissipated entirely.

In running the body falls ballistically until the feet land on the ground. Then the legs deform elastically to absorb the body's kinetic energy, and they return the energy a short time later to help power the next step. Although a passively bouncing ball must eventually come to rest, a legged system can sustain its oscillation indefinitely by using leg actuators to replace lost energy. Margaria used this bouncing-ball model of running to distinguish running from walking, which, he points out, is better modeled by the rolling of an egg (Margaria 1976).

Elastic storage and recovery of energy is particularly important to the efficiency of the hopping kangaroo (Dawson and Taylor 1973), but it also helps other animals, including the human, to run efficiently (Cavagna et al. 1977, Alexander and Jayes 1978). Perhaps more important than the efficiency of bouncing is the simplicity bouncing gives to the control of the locomotion cycle. Hopping can rely on the passive bouncing oscillation to generate the detailed pattern of the motion, while the control system excites the oscillation and regulates its amplitude.

In addition to bouncing like a ball, running systems tip like an inverted pendulum. An inverted pendulum has an elevated mass that pivots above a support point. When the mass is located directly over the support point, there are no tipping moments, so the system is in equilibrium. Any small displacement of the mass from directly over the support point, however, causes tipping moments that drive the pendulum further from equilibrium. The equilibrium point is unstable. A control system can provide balance for an inverted pendulum by moving the support point back and forth in response to tipping motions. Actually, the balanced inverted pendulum tips all the time, but the control system keeps it from tipping over entirely by ensuring that each tipping motion in one direction is balanced by an equal and opposite tipping motion in the other direction.

Legged systems behave like this, too; they tip about unstable equilibria when their feet are not directly under the body, and they balance by moving their feet in response to tipping motions. Unfortunately, several factors complicate the story for legged systems. For instance, legs usually change length when they are loaded by the body, so the distance between the mass and the support point varies. Also, legs often have large feet that cause the instantaneous support point to move during tipping. Perhaps most important, a system that runs can move the pivot points only when the legs are unloaded during flight. Despite these complications, the concept of an inverted pendulum and our knowledge of its behavior greatly simplifies thinking about balance in running.

In this chapter I describe a machine that runs by hopping on a single springy leg. It can bounce like a ball and tip like an inverted pendulum, making it an ideal vehicle for the study of mechanisms underlying running. With only one leg there is no need to coordinate several legs, so this difficult problem is avoided, whereas the need for active balance is central.

The control system for running that my research group has explored decomposes the control task into three separate parts that regulate hopping, forward travel, and posture. This three-part control system permits the one-legged machine to hop in place, run at a desired rate, travel from place to place, maintain its balance when disturbed mechanically, and leap over small obstacles. Top running speed is 1.2 m/s (2.6 mph). This first hopping machine is restricted to move in the plane, but a three-dimensional version is considered in the next chapter.

A Planar Machine That Hops on One Leg

Figure 2.1 shows the machine we used to study running. Its main parts are a body and a leg, connected by a hinge-type hip. It has only one leg,

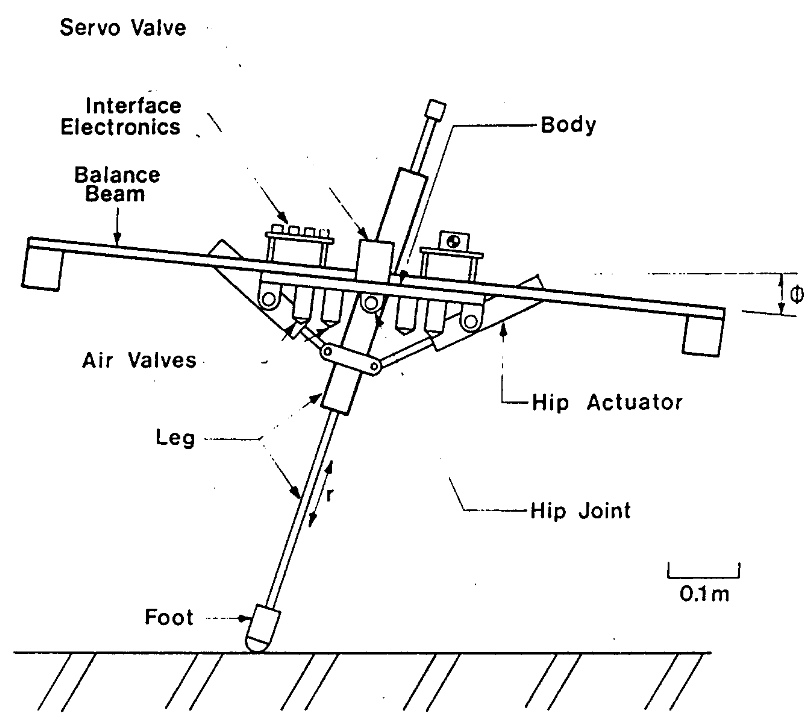


Figure 2.1. Planar one-legged hopping machine. It has two main parts: a body and a leg. The body provides mounting for valves, electronics, and sensors, and it has a weighted beam for increased moment of inertia. The leg is an air cylinder that pivots with respect to the body, with a padded foot at one end. The machine is powered by compressed air. Four on-off solenoid valves control the flow of air to and from the leg cylinder. They can trap air in the leg cylinder to make it act like a spring. A pair of pneumatic actuators exerts a torque between the leg and the body about the hip. These actuators are powered by a proportional pressure-control servo valve. Sensors mounted on the machine measure the length of the leg, the angle of the hip, contact between the foot and the ground, and the pressures in the leg air cylinder.

so hopping is the only gait it can use. The leg is springy, enabling it to use a resonant oscillation for hopping. The body consists of a platform that carries sensors, valves, actuators, and computer interface electronics.

The behavior of the hopping machine was simplified by restricting its motion to the plane. The tether mechanism shown in figure 2.2 constrains the machine to move with just three degrees of freedom; it can move fore and aft and up and down and can rotate about the pitch axis. The tether

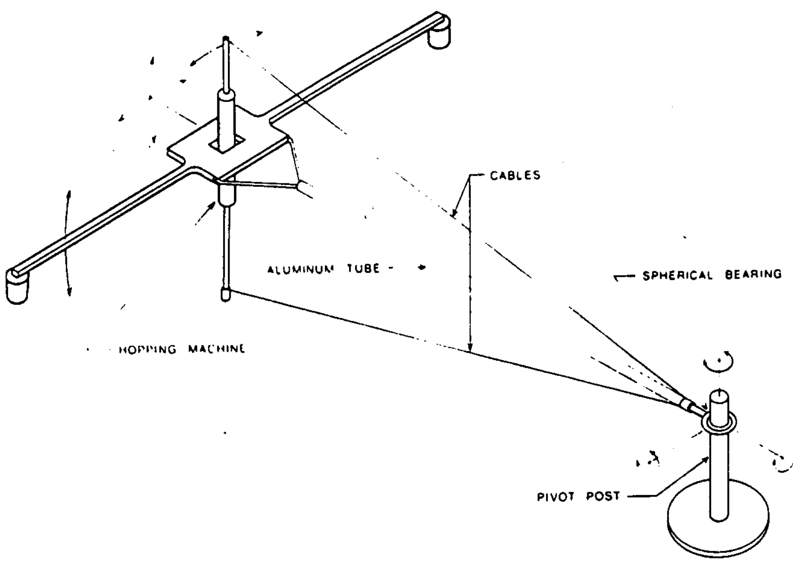


Figure 2.2. The tether mechanism constrains motion of the hopping machine to three degrees of freedom, permitting it to travel on a large circle in the laboratory. The mechanism consists of an aluminum tube, a spherical pivot fixed to the floor, a fork pivot fixed to the hopping machine, and tension cables. This arrangement keeps the machine 2.5 m from the fixed spherical pivot, giving it radial and yaw stability. A pair of nylon cables prevents motion about the roll axis. The cables also keep the foot a nearly constant distance from the spherical pivot as the leg changes length, minimizing radial scrubbing. The tether is instrumented to provide measurements of the machine's three motions: vertical translation, forward translation, and rotation about the axis of the boom.

prevents lateral translation, roll rotation, and yaw rotation. Actually the machine moves on the surface of a large sphere centered at the tether pivot. In earlier experiments the machine was constrained by air bearings that let it float on an inclined table, making its motion truly planar. But when the machine began to run at speed, it quickly traveled the full length of the table, and fell off the end. The tether permits the machine to travel on a continuous circular path with a radius of 2.5 m. No changes in the control were needed to convert from planar to spherical operation.

Sensors mounted on the tether's pivoting base measure the machine's forward position on the circle and the pitch angle of the body. The tether supports an umbilical cable that connects the hopping machine to a source

of compressed air, electrical power supplies, and the control computer.

The body and leg are connected by a hinge joint that forms a hip. A proportional pneumatic pressure-control valve drives a pair of air cylinders that exerts torques about the hip. A potentiometer measures the angle between the body and the leg, the hip angle γ. The control computer servos the hip angle with a simple linear servo:

position damping

$$\tau = -k_p(\gamma - \gamma_d) - k_v(\dot{\gamma}), \tag{2.1}$$

stiffness velocity

where

τ	is the actuator torque generated at the hip,
γ	is the hip angle,
γ_d	is the desired hip angle,[1] and
k_p, k_v	are position and velocity feedback gains. Typical values are $k_p = 47\mathrm{N} \cdot \mathrm{m/rad}$, and $k_v = 1.26\,\mathrm{N} \cdot \mathrm{m/(rad/s)}$.

A full 40° sweep of the leg takes approximately 120 ms with a servo rate of 500 hz. The ratio of the moment of inertia of the body to that of the leg is 14:1. This relatively high ratio ensures that the orientation of the leg can change during flight without severely disturbing the attitude of the body. The center of mass of the body is located at the hip, so the only moments acting on the body are those generated by the hip actuator. Table 2.1 gives dimensions and parameters for the machine.

The Leg

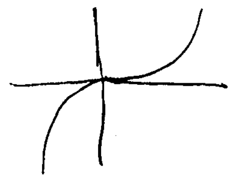
The leg consists of a double-acting air cylinder with a padded foot attached to the lower end of the cylinder rod. The foot is narrow, about 20 mm when fully loaded, providing a good approximation to a point of support. The coefficient of friction between the foot and the floor in our laboratory is about 0.6, so the foot does not slip much. The foot has a switch in it that closes whenever it touches the floor. A Rube Goldberg arrangement of aircraft wire, a pulley, and a potentiometer provides a measurement of leg length r, the distance from the hip to the foot.

Four electric solenoid valves control the flow of compressed air to the leg's air cylinder (figure 2.3). The valves connect each chamber of the leg cylinder either to atmospheric pressure through a flow-restricting orifice or to a regulated supply at 90 psi. Sensors monitor the air pressures in both chambers. Delivery of pressurized air to the top chamber of the cylinder

[1] Throughout this book a variable with subscript d specifies the variable's desired value.

drives the piston and rod assembly downward, providing a vertical thrust for hopping.

The leg is made springy by trapping air in the upper chamber of the leg cylinder, with both solenoid valves closed. It is possible to regulate the air pressure in a chamber by charging it to a higher pressure than desired, then exhausting air through the flow-restricting orifice until the desired pressure is reached, and then closing the exhaust valve. The solenoid valves operate in about 10 ms, resulting in pressure regulation to about 1 psi. When the leg shortens under load, the trapped air compresses, acting like a $1/r$ spring. The effective stiffness of the spring is determined by the resting pressure in the chamber.



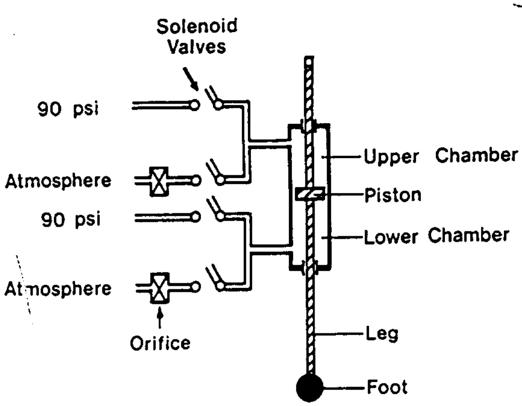


Figure 2.3. The leg actuator is a pneumatic cylinder. Electric solenoid valves control the flow of air to both chambers of the cylinder. When both valves to a chamber are off, trapped air makes the leg springy. Pressure sensors (not shown) measure the air pressure in both chambers.

Operation of the Machine

To initiate hopping, someone has to drop the machine from a shallow height onto the extended leg. Then the control system operates the solenoid valves to excite and sustain the hopping motion. The control system applies thrust by opening the supply solenoid valve to the upper chamber of the leg cylinder during each stance. The duration of this valve's operation is defined as the magnitude of thrust. Once the foot leaves the ground, the control system exhausts the upper chamber of the leg cylinder until it reaches a designated pressure, typically 15 psi. Increasing the pressure during stance

Table 2.1. Physical parameters of planar one-legged hopping machine.

Parameter	Metric Units	English Units
Overall height	0.69 m	27.3 in
Overall width	0.97 m	38.0 in
Hip height	0.5 m	19.5 in
Total mass	8.6 kg	19 lbm
Unsprung leg mass	0.45 kg	1.0 lbm
$\frac{\text{Body mass}}{\text{Unsprung leg mass}}$	18:1	18:1
Body moment of inertia	$0.52\,\text{kg}\cdot\text{m}^2$	$1770\,\text{lbm}\cdot\text{in}^2$
Leg moment of inertia	$0.037\,\text{kg}\cdot\text{m}^2$	$125\,\text{lbm}\cdot\text{in}^2$
$\frac{\text{Body moment of inertia}}{\text{Leg moment of inertia}}$	14:1	14:1
Leg Axial Motion		
Stroke	0.25 m	10.0 in
Static force	360 N @ 620 kPa	80 lb @ 90 psi
Leg Sweep Motion		
Sweep angle	± 0.33 rad	$\pm 19°$
Static torque	$27\,\text{N}\cdot\text{m}$ @ 620 kPa	$240\,\text{lb}\cdot\text{in}$ @ 90 psi

and decreasing it during flight excites the spring-mass/gravity-mass oscillator formed by the leg and the body. Peak to peak amplitude of body oscillation can be varied between 0.04 and 0.3 m, with corresponding hopping frequencies of 3 and 1.5 hops per second. Over this range of frequencies the duration of stance is nearly constant at about 175 ms, with just a few percent variation.

During the hopping cycle, accelerations of the unsprung part of the leg dissipate a fraction of the hopping energy. The mass of the unsprung part of the leg is m_ℓ, and the remaining mass of the system is m. From conservation of linear momentum we find that $m_\ell/(m_\ell+m)$ of the hopping energy is lost each time the foot strikes the ground and each time the foot leaves the ground. This assumes that collisions between the foot and the ground and collisions of the piston with the leg cylinder are plastic, with a coefficient of restitution of zero. The ratio of body mass to unsprung mass in the planar one-legged hopping machine is 18:1, resulting in an 11% energy loss for each hopping cycle. Other losses are due to friction in the leg cylinder. Under ideal testing conditions, frictional losses dissipate about 25% of the hopping energy on each bounce.

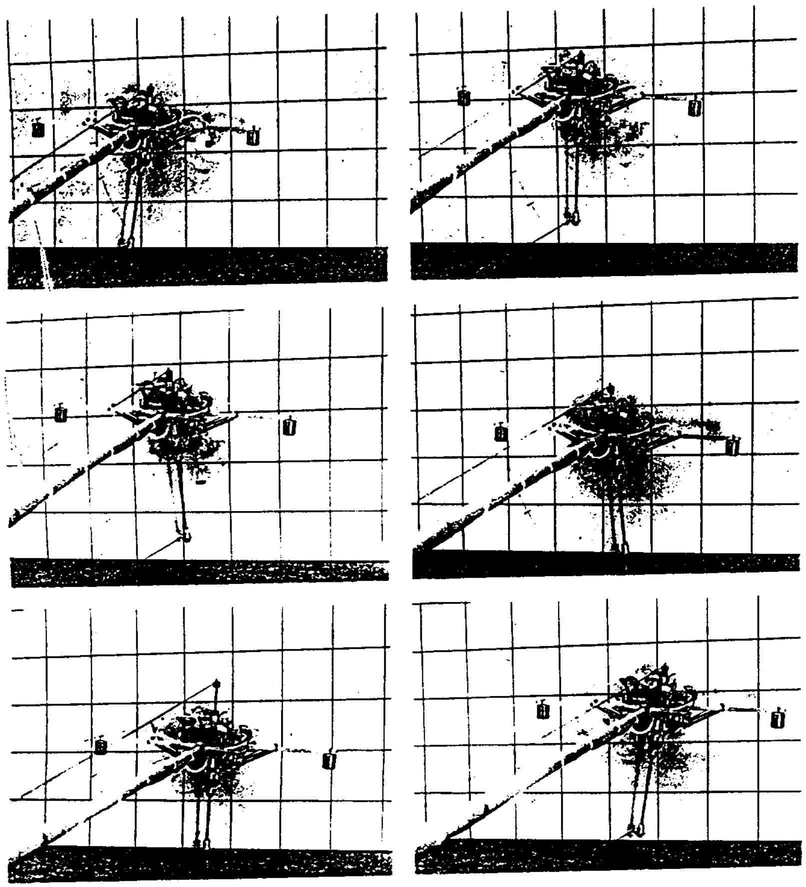


Figure 2.4. One complete stride of hopping at a speed of 0.75 m/s. Stride length was 0.45 m, stride period 0.68 s. Background grid spacing is 0.2 m. Adjacent frames separated by 100 ms. From Raibert and Brown (1984).

Figure 2.4 illustrates the general operation of the machine as it hops. The leg actuator drives the vertical bouncing motion. The control system extends the leg forward during flight according to the rate of forward travel the faster it is going, the further forward it extends the leg. The control system also servos the hip during stance to keep the body upright. There are four events in this hopping cycle that are useful to name:

1. *Lift-off.* The moment the foot loses contact with the ground.
2. *Top.* The moment in flight when the body has maximum altitude and vertical motion changes from upward to downward.
3. *Touchdown.* The moment the foot makes contact with the ground.
4. *Bottom.* The moment during stance when the body has minimum altitude and its vertical velocity changes from downward to upward.

Control of Running Decomposed into Three Parts

The control system we explored for the planar one-legged machine treats hopping, forward speed, and body attitude as three separate control problems. One part of the control excites the hopping motion and regulates its amplitude by specifying the thrust to be delivered by the leg on each hop. The second part of the control stabilizes the machine's forward speed by extending the foot forward to a position that will provide the needed acceleration during stance. The third part of the control maintains the body in an upright attitude by servoing the hip during stance. These three parts of the control system are synchronized by a finite state machine that tracks the machine's hopping activity. By decomposing the problem in this way we rely on a weak coupling between these motions.

Control Hopping Height

In order for a legged system to operate and to make forward progress, each leg must spend some of its time supporting the weight of the body and some of its time unloaded, with the foot free to move. An alternation between a loaded phase and an unloaded phase is observed in the legs of all legged systems. For the one-legged machine the loaded phase is an elastic collision and the unloaded phase is ballistic flight, just like the bouncing ball mentioned earlier. The overall hopping behavior is an oscillation that is largely passive, with the details of the motion determined by the springiness of the leg, the mass of the body, and gravity.

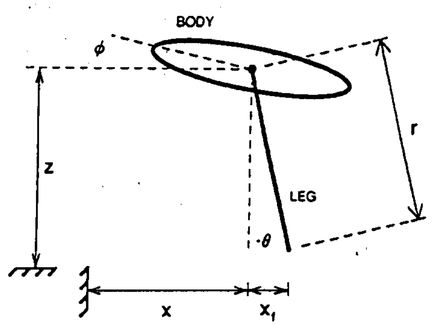


Figure 2.5. Diagram of planar one-leg machine showing variables used for control.

The control system relies on this passive mechanical oscillation to determine the form of the basic hopping motion, whereas leg thrust delivered to the body during each hop determines the amplitude. In principle, the control system could figure out how hard to thrust by comparing the energy needed to reach a desired hopping height with the actual energy, making up the difference with leg thrust. Such a calculation could take into account the kinetic energy of the body, the elastic energy of the leg spring, and the expected energy loss. This approach was quite effective in controlling a hopping model studied by computer simulation (see chapter 6), but a simpler method is used here.

The control system for the hopping machine delivers a fixed thrust during each stance phase. This causes the bouncing motion to come to equilibrium at a hopping height for which the energy injected by thrust just equals the energy lost to friction and accelerating unsprung leg mass. Because these mechanical losses are monotonic with hopping height, a unique equilibrium hopping height exists for each fixed value of thrust, and greater thrust results in greater height. The relationship between thrust and hopping height is not simple. The operator is left with the task of choosing a fixed value for thrust that results in an acceptable hopping height during a set of experiments.

Hopping data recorded from the physical one-legged machine are plotted in figure 2.6. A new thrust was specified every 5 seconds while the machine hopped in place. Each time the setpoint changed, it took the machine four or five hops for the hopping amplitude to stabilize. Data from four cycles of level hopping are replotted in the form of a phase diagram in figure 2.7. The four trajectories overlap precisely in this figure, indicating that the hopping motion was stable. The slight indentation just after

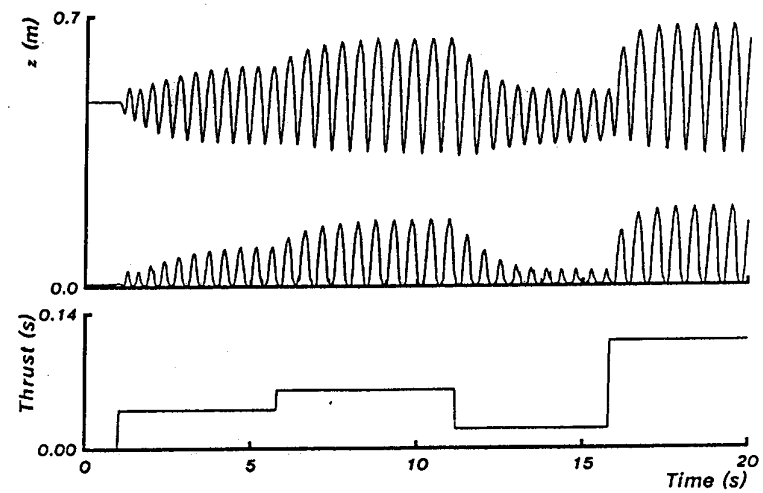


Figure 2.6. Data recorded while the machine hopped in place. Every 5 seconds the duration of vertical thrust was adjusted to change hopping height. In each case it took about 2 seconds and four cycles to arrive at equilibrium. (Top curve) elevation of the hip, z; (middle curve) elevation of the foot, $z - z_f$; and (bottom curve) duration of thrust. From Raibert and Brown (1984)

lift-off is due to the sudden deceleration of the body that occurs when the leg is accelerated to body speed. The trajectory is parabolic in the upper part of this diagram because of gravity's constant acceleration and nearly harmonic in the lower part because of the leg spring. Because the leg spring is not linear—it has a $1/r$ characteristic that makes it a *hard* spring—there is a slight deviation from harmonic behavior.

A State Machine Tracks the Hopping Cycle

An important function of the hopping motion is to provide a regular cycle of activity that synchronizes the control. A state machine keeps track of the hopping motion by switching state when sensory data indicate the occurrence of key events. A new set of control actions takes effect during each state. For example, the state machine switches from COMPRESSION to THRUST when the derivative of leg length changes from negative to positive ($\dot{r} > 0$). The action taken is to begin leg thrust and to servo the body attitude. Figure 2.8 shows the cycle of activity used for one-legged hopping, and table 2.2 provides some additional details.

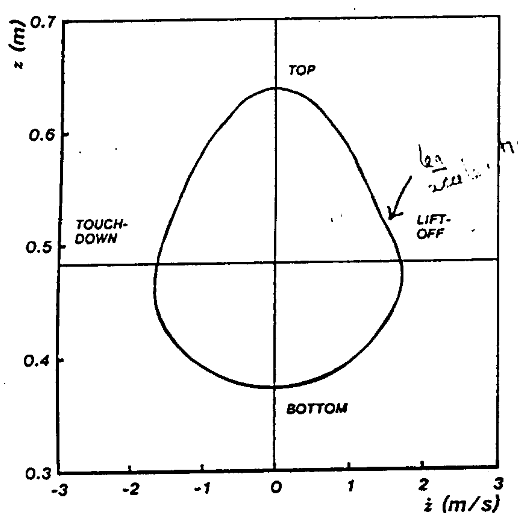


Figure 2.7. Phase plot of vertical hopping. Four cycles of level hopping at a fixed height are replotted in the phase plane. The curves cross the axes at lift-off, top, touchdown, and bottom. Note. Unlike a normal phase plot, position is plotted on the ordinate and velocity on the abscissa, so time progresses in the counterclockwise direction. From Raibert and Brown (1984).

Control Forward Speed

The position of the foot when it first touches the ground at the end of flight has a powerful influence on the accelerations that occur during the ensuing stance phase. The accelerations are like those of an inverted pendulum in that the foot's position with respect to the center of mass determines the tipping moments. In addition, the forward speed of the body influences the accelerations, as do the vertical speed and the axial leg force.

The control system manipulates these accelerations to control forward running speed by choosing a forward position for the foot before each landing. Because the leg is connected to the body, the control system can position the foot with respect to the body any time during flight in order to determine the relative position at touchdown. Once the foot is in position and the stance phase begins, the control system takes no further action for the remainder of the step—the dynamics of the mechanical system consisting of the body, the leg, and the ground govern what happens. For a wide range of conditions, the *net forward acceleration*, the difference between forward speed at touchdown and at lift-off, $\Delta \dot{x} = \dot{x}_{lo} - \dot{x}_{td}$, is a

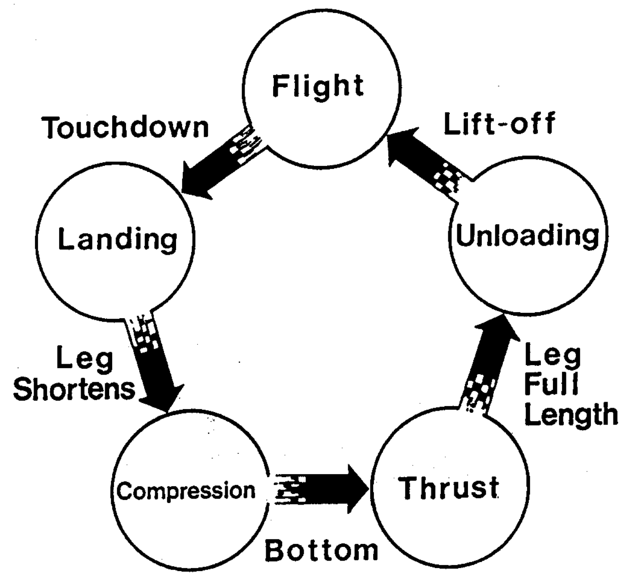


Figure 2.8. A state machine tracks the hopping behavior to synchronize the three parts of the control system. Sensory information triggers transitions between the states, and each state specifies what the control system should do.

Table 2.2. Details of state machine sequence for the hopping cycle. The state shown in the left-hand column is entered when the event in the center column occurs. The control action to be taken is shown in the column on the right. States advance sequentially during normal hopping. States LOADING and UNLOADING help to isolate the stance and flight phases from each other, as described in the text.

State	Trigger Event	Action
1 LOADING	Foot touches ground	Stop exhausting leg Zero hip torque
2 COMPRESSION	Leg shortens	Upper leg chamber sealed Servo body attitude with hip
3 THRUST	Leg lengthens	Pressurize leg Servo body attitude with hip
4 UNLOADING	Leg near full length	Stop thrust Zero hip torque
5 FLIGHT	Foot not touching	Exhaust leg to low pressure Position leg for landing

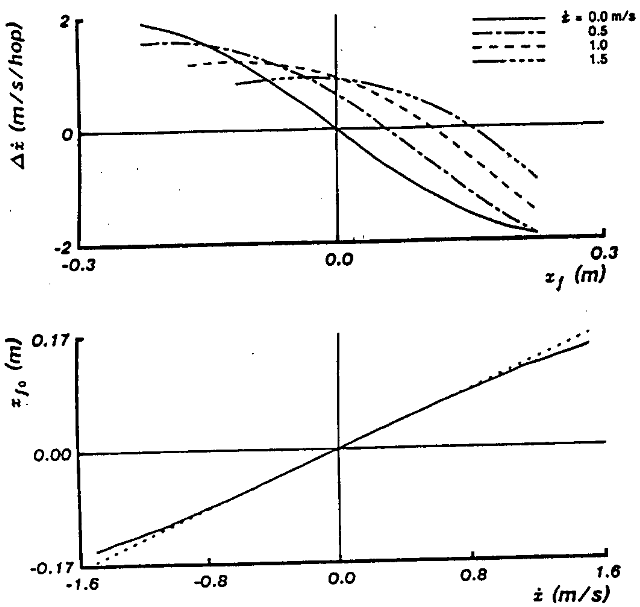


Figure 2.9. (Top) The net forward acceleration varies with forward foot position. The parameter is forward speed. Data are from simulations of a one-legged hopping machine with a linear leg spring. (Bottom) The location of the neutral point varies with forward running speed. The function is nearly linear up to about 1 m/s.

monotonic function of the forward position of the foot at touchdown. The net forward acceleration is a single number that summarizes accelerations that occur throughout the stance phase. It has units of m/s/hop. The forward acceleration is assumed to be zero throughout flight,[2] so accelerations during the stance phase control speed.

For each forward speed there is a unique foot position that results in zero net forward acceleration. We call this the *neutral point*, designated x_{f0}. For hopping in place with no forward travel, the neutral point is located directly under the body, but for nonzero forward speed it is located in front of the body in the direction of travel. The faster the running the further forward the neutral point, as shown in figure 2.9.

[2] Throughout this book I ignore the effects of air resistance during both stance and flight. See Pugh (1971) for measurements of wind drag on humans in walking and running.

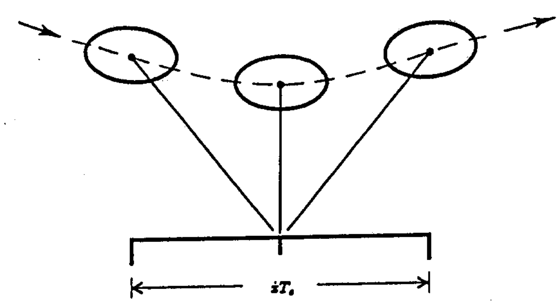


Figure 2.10. Symmetric trajectory. When the foot is placed in the neutral position, there is a symmetric motion of the body with respect to the foot. The figure shows the configuration just before the foot touches the ground (left), the configuration halfway through stance when the leg is vertical and maximally compressed (center), and the configuration just after the foot loses contact with the ground (right). The forward position of the body, the angle of the body, and the angle of the leg have odd symmetry, $x(t) = -x(-t)$, $\phi(t) = -\phi(-t)$, $\theta(t) = -\theta(-t)$, whereas the vertical position of the body and leg length have even symmetry, $z(t) = z(-t)$, $r(t) = r(-t)$. Time and position are defined so that $t = 0$ halfway through the stance phase, and $x(0) = 0$. The locus of points over which the center of gravity travels during stance is called the *CG-print*. It is shown by the horizontal bar at the bottom of the diagram.

Symmetry and Asymmetry

When the foot is placed on the neutral point, the body's center of mass travels over the foot during stance with a symmetric motion described by even and odd functions of time. The schematic in figure 2.10 shows this kind of symmetric behavior. When the system moves with symmetry, the center of mass spends the same amount of time in front of the foot as it spends behind the foot, so forward tipping that occurs during the second half of stance precisely compensates for the backward tipping that occurs during the first half of stance. The horizontal components of the axial leg force also balance because the leg is maximally compressed at the same time the foot is located under the center of mass. This assumes that the axial leg force $f(t)$ is an even function of time during stance, which would be the case if the vertical bouncing motion of the body on the leg were passive with neither losses nor thrust. The forward speed does not change because the horizontal forces acting on the body throughout the stance phase average to zero. Another way to say this is that for a symmetric body motion the tipping moments and horizontal ground forces are odd functions of time during stance. Odd functions integrate to zero over symmetric limits, providing zero net acceleration.

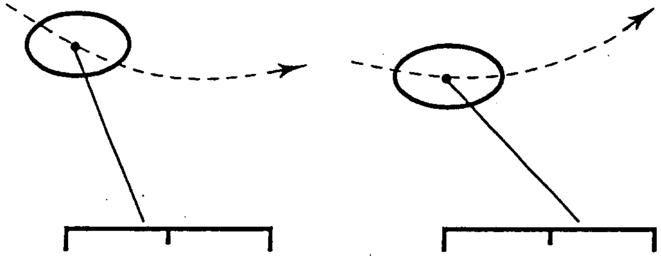


Figure 2.11. Asymmetric trajectories. Displacement of the foot from the neutral position accelerates the body by skewing its trajectory. When the foot is placed behind the neutral point, the body accelerates forward during stance (left). When the foot is place forward of the neutral point, the body accelerates backward during stance (right). Dashed lines indicate the path of the body, and solid horizontal lines under each figure indicate the CG-print.

Displacement of the foot from the neutral point results in body trajectories that are no longer symmetric, as illustrated in figure 2.11. They are skewed according to the sign and magnitude of the foot displacement. The skewed trajectories have nonzero net forward acceleration of the body, and the forward speed changes as a result. By placing the foot forward of the neutral point, the control system creates a net rearward acceleration that slows the machine down. By placing the foot behind the neutral point it creates a net forward acceleration that speeds the machine up. Figure 2.9 shows the functional relationship between the net forward acceleration and displacement from the neutral point. The relationship is nearly linear for small displacements at a single forward speed. Figure 2.12 shows the path of the body during stance for the different foot positions described.

An Algorithm for Foot Placement

To regulate forward speed, the control system must calculate a forward position for the foot based on the state of the machine and the desired behavior. There are several ways to solve this problem. One is to solve the equations of motion for the system to find expressions for the state variables as functions of time. These solutions could be inverted to express foot position as a function of state and desired behavior. A control system could plug the present and desired states of the system into such closed-form solutions to calculate the required forward foot placement. Unfortunately, analytic solutions to the differential equations that describe mechanical systems are known only rarely and in most cases analytic solutions do not even exist. Closed form expressions relating forward foot placement to net

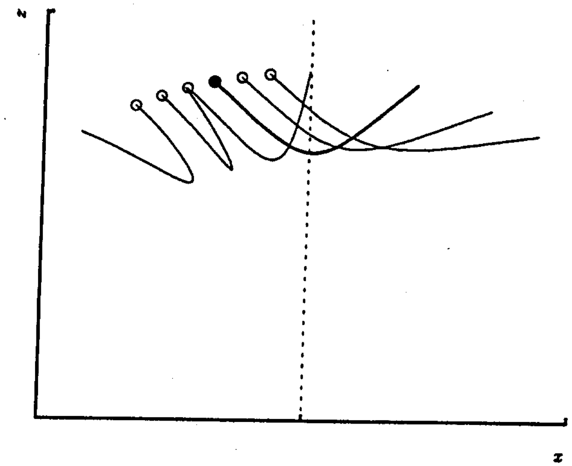


Figure 2.12. Path of the body during stance for several forward foot positions. Only the neutral foot position results in a symmetric body trajectory (bold), whereas those to either side are skewed, either forward or backward. The initial forward speed is the same for each trajectory. The circles indicate the location of the body at touchdown, and the origin is the foot position. These data are from simulations of a model with a linear leg spring. Adapted from Stentz (1983).

forward acceleration for the one-legged machine are not known.

A second approach would be to simulate numerically a large enough set of situations so that the results could be tabulated to provide approximate solutions. We have explored this technique for a simple legged model with encouraging results that are described in chapter 7.

A third approach, the one taken here, is to use closed-form approximations to the solutions. The control system we implemented uses rather crude but simple approximations to estimate the location of the neutral point and to choose a forward position for the foot. Despite several shortcomings, these approximations have proven to be quite effective.

Two factors enter into the calculation of forward foot position as implemented. The measured forward speed is used to approximate the location of the neutral point. The error in forward speed is used to calculate a displacement from the neutral point to accelerate the system. The neutral point and the displacement combine to specify how the control system places the foot.

To calculate the neutral point, the control system estimates the locus of points over which the center of gravity will travel during the next stance phase. We call this locus the *CG-print*; it is analogous to a footprint. From figure 2.10 we see that the center of the CG-print is the neutral point. With the foot located in the center of the CG-print, the values of leg angle and forward body position at touchdown are equal but opposite in sign to their values at lift-off. This satisfies the symmetry described earlier. The length of the CG-print is approximately the product of the forward speed and the duration of stance, $\dot{x}T_s$. To place the foot in the center of the CG-print, the control system extends the leg forward during flight so that the foot is a distance in front of the hip:

$$x_{f0} = \frac{\dot{x}T_s}{2}, \tag{2.2}$$

where

x_{f0}	is the forward displacement of the foot with respect to the the center of mass,
$\dot{x}$	is the forward speed, and
T_s	is the duration of the stance phase.

Because a spring mass system oscillates with a period that is independent of amplitude, the duration of the stance phase is nearly constant for a given leg stiffness. The control system uses the duration of the previous stance phase as the expected duration for the next stance phase. To the extent that the body continues to travel forward during stance with speed $\dot{x}$ and to the extent that the compression of the leg has even symmetry, (2.2) places the foot at the neutral point to provide unaccelerated travel.

To accelerate the machine the control system introduces asymmetry. Acceleration is needed to stabilize the forward speed against errors and external disturbances and to change from one forward speed to another. To accelerate the machine on purpose, the control system displaces the foot from the neutral point (see figure 2.11). The control system uses a linear function of the error in forward speed to find a displacement for the foot:

$$x_{f\Delta} = k_{\dot{x}}(\dot{x} - \dot{x}_d), \tag{2.3}$$

where

$x_{f\Delta}$	is the displacement of the foot from the neutral point,
$\dot{x}_d$	is the desired forward speed, and
$k_{\dot{x}}$	is a feedback gain.

Combining (2.2) and (2.3) yields the algorithm for placing the foot:

$$x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d). \tag{2.4}$$

Once the control system calculates x_f, kinematics are used to find the required hip angle (see figure 2.5):

$$\gamma_d = \phi - \arcsin\left(\frac{\dot{x}T_s}{2r} + \frac{k_{\dot{x}}(\dot{x} - \dot{x}_d)}{r}\right). \tag{2.5}$$

where γ is the angle between the leg and the body. The servo given by (2.1) drives the hip joint. This algorithm for placing the foot controls forward speed and accelerations when hopping in place, accelerating to a run, running with constant speed, and slowing to a stop. The process of choosing a foot position to control acceleration is the primary mechanism used for balance.

Control Body Attitude

The control system maintains an upright body attitude by exerting torques about the hip during stance. Because angular momentum is conserved during flight, the stance phase provides the only opportunity to change the angular momentum of the whole system. Friction between the foot and the ground during stance permits torques to be applied to the body without causing large accelerations of the leg. These torques are used to servo the body to the desired attitude. The control system does this with a linear servo:

$$\tau = -k_p(\phi - \phi_d) - k_v(\dot{\phi}), \tag{2.6}$$

where

τ	is the hip torque,
ϕ	is the pitch angle of the body, and
k_p, k_v	are position and velocity feedback gains. Typical values are $k_p = 153\,\mathrm{N\cdot m/rad}$, and $k_v = 14\,\mathrm{N\cdot m/(rad/s)}$.

Friction keeps the foot from slipping on the ground. Its magnitude is proportional to the normal force. Precautions were taken to ensure that there is adequate normal force to hold the foot in place when hip torques are used to correct the body attitude during stance. Two states, LOADING and UNLOADING, were added to the state machine that synchronizes the control system to the hopping behavior. These states prevent the body attitude servo from operating when the leg just begins to accept load after touchdown and when it is nearly unloaded just before lift-off. I think of these as *twilight states* because they suggest that the machine is neither fully in stance nor fully in flight. Another twilight state, ESCAPE (not shown in the diagram), keeps the control system from moving the leg forward just after lift-off until the foot has attained sufficient altitude to clear the ground. Premature movement of the leg would stub the toe.

To summarize this section, the control system operates as three separate parts. One part regulates the hopping motion by delivering a thrust to the body during each support phase. The second part of the control manipulates forward speed by choosing a position for the foot that will provide the required net forward acceleration during the next stance phase. The third part servos the body to an upright posture when friction holds the foot in place during stance. We now turn to experiments to evaluate the approach.

Hopping Experiments

The one-legged hopping machine was used to explore the workability of the three-part control decomposition and to demonstrate balance in a dynamic legged machine. The algorithms for hopping, forward speed, and body attitude and the finite state machine were implemented in a set of computer programs that ran on a minicomputer. These programs controlled the hopping machine and recorded its behavior.

Rate Control

To test regulation of forward speed, the control computer specified a staircase of desired values over a 10-second interval. Before the interval began, the machine hopped in place, with desired forward speed specified from a joystick. The results from the test are plotted in figure 2.13. The machine started by hopping in place, then increased speed up to about 0.9 m/s, then held speed, and finally came to a stop. Throughout the test, error in forward speed was controlled to about ± 0.25 m/s. This accuracy is typical. It was possible to improve the regulation of forward speed at any given speed by adjusting the velocity error gain, $k_{\dot{x}}$ in (2.4). The need for this adjustment has already been suggested by figure 2.9, which shows that the relationship between foot displacement and net forward acceleration depends on forward speed.

During running the leg and body counteroscillate, as shown in the plots of θ and ϕ in figure 2.13. Oscillations of the body are expected because angular momentum must be conserved during flight and because

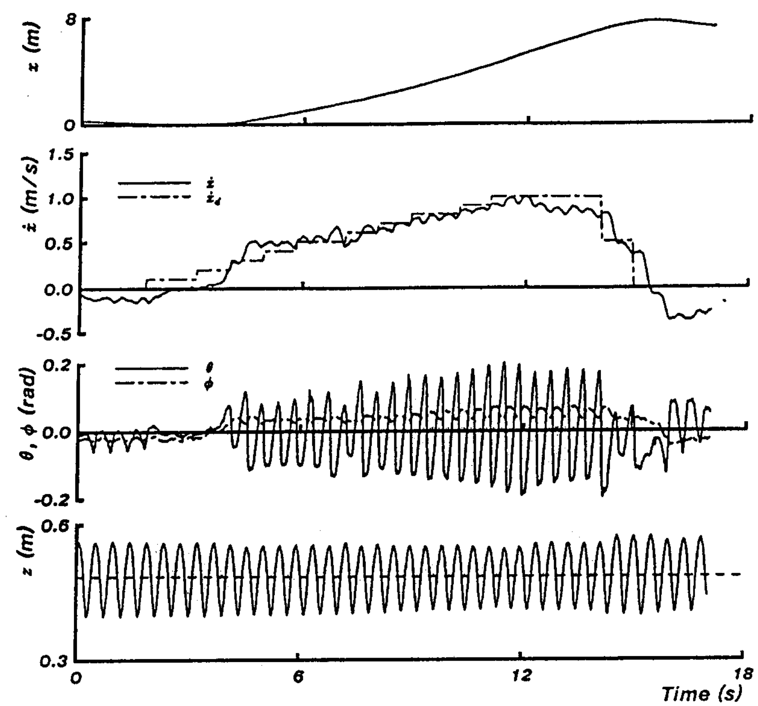


Figure 2.13. Control of forward running speed was tested by varying $\dot{x}_d$, the rate setpoint (dashed-dotted line), from 0 to 1.0 m/s over a 10 s period. Also shown are the forward position of the machine x, the body pitch angle ϕ, the vertical position of the body z, and the leg angle θ. The dashed line on the plot of z (bottom curve) separates stance (below line) from flight (above line). From Raibert and Brown (1984).

the attitude of the body is corrected only during stance. The average body pitch angle deviates from zero in rough proportion to running speed, as indicated by the plot of ϕ in figure 2.13. Hopping height and stride frequency are also affected by running speed. Actually, the relevant factor is not running speed directly but the angle of the leg at touchdown. Faster running results in large deviations of the leg from vertical and therefore shallower hops. These shallower hops have shorter flight time and result in more rapid stepping. When the machine runs at 0.9 m/s, the peak foot clearance is reduced by 20% and stride period is reduced by 8.6%.

During running the leg sweeps back and forth like the legs of running animals. In the case of the hopping machine, these motions were not explicitly programmed. They emerged as a by-product of the interplay between the control of forward speed, which moves the foot to a forward position during flight, and the control of body attitude, which permits the body to coast past the foot during stance.

Position Control

Position control was used to make the machine stay in one place or to translate from place to place. A position controller was built on top of the three-part control system by transforming position errors into desired forward speeds:

$$\dot{x}_d = \min\left[k(x - x_d), \dot{x}_{max}\right], \tag{2.7}$$

where x_d is the target location and $\dot{x}_{max}$ limits the maximum rate of travel when the machine is far from the target. Target positions were sometimes specified by hand with a joystick and sometimes by the control computer according to a programmed sequence. Data obtained from the machine under position control are plotted in figure 2.14. The machine stayed on the specified location with less than ± 0.1 m error.

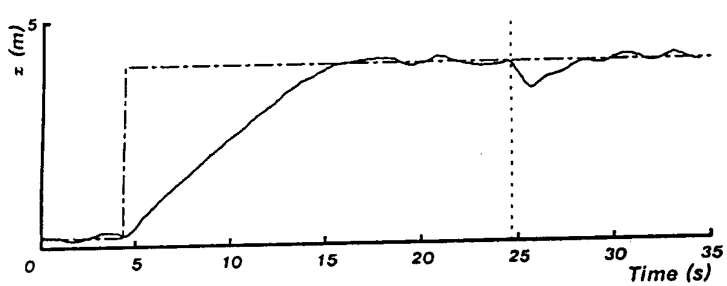


Figure 2.14. Position control. Position errors were transformed into rate setpoints to control the machine's forward position. After 4.3 seconds of stationary hopping, the computer specified a 4-m change in desired position (dot-dashed line). A limit cycle of about ± 0.1 m is present whenever the control system keeps the machine in one place. The experimenter disturbed the machine by delivering a sharp horizontal jab by hand (vertical dotted line). It returned to the setpoint within a few seconds. From Raibert and Brown (1984).

Also shown in figure 2.14 is the response to an external mechanical disturbance. After about 25 seconds the experimenter delivered a sharp horizontal jab to the body as the machine hopped in place. The machine recovered its balance and returned to the commanded position after a few seconds. The control system tolerated fairly strong disturbances, provided

that the forces were primarily horizontal. Disturbances that introduced large rotations of the body usually caused the machine to tip over.

Leaping

A specialized version of the hopping part of the control system makes the machine leap. To demonstrate leaping, the machine approached a small obstacle with a moderate running rate. One step before the obstacle the operator pressed a *leap button*, initiating a preplanned sequence that was synchronized to vertical hopping by the state machine. The sequence began at the start of the next stance phase:

1. Thrust is delayed so that the leg shortens more than normal under load of the body. This is done to prepare for a hop of maximum height. Once thrust begins, it continues until the leg extends fully.
2. Once airborne, the leg shortens, and the sweeping motion is delayed; both provide extra clearance for the foot.
3. At the peak of the hop the leg swings to the correct landing angle. There is less time to swing the leg than normal, but the shorter leg moves more quickly because of the reduced moment of inertia.
4. The leg lengthens in preparation for landing.
5. Upon landing, the standard hopping sequence is re-established.

The control system uses the standard forward speed and body attitude algorithms during leaping.

This procedure was used to leap over stacks of styrofoam blocks, as shown in figure 2.15. Although many leaps were successful, at least as many were failures. The general task of jumping over an obstacle requires that the foot be placed in a suitable location on the ground relative to the obstacle on the step before the leap, that the leap have sufficient altitude, and that the leap have sufficient span. The existing control system does a good job with height and span, but it cannot manipulate the takeoff point.

The task of placing the foot on a particular location is more demanding than merely controlling the forward speed as the existing control system does. In order to position the foot for takeoff, it is necessary to control the D. Lee stride during a number of steps before the leap. This can be accomplished by adjusting the forward speed while approaching the obstacle, by adjusting leg stiffness, or by adjusting the height of each hop. The need to position feet on specific footholds is an important part of the larger problem of traveling on rough terrain.

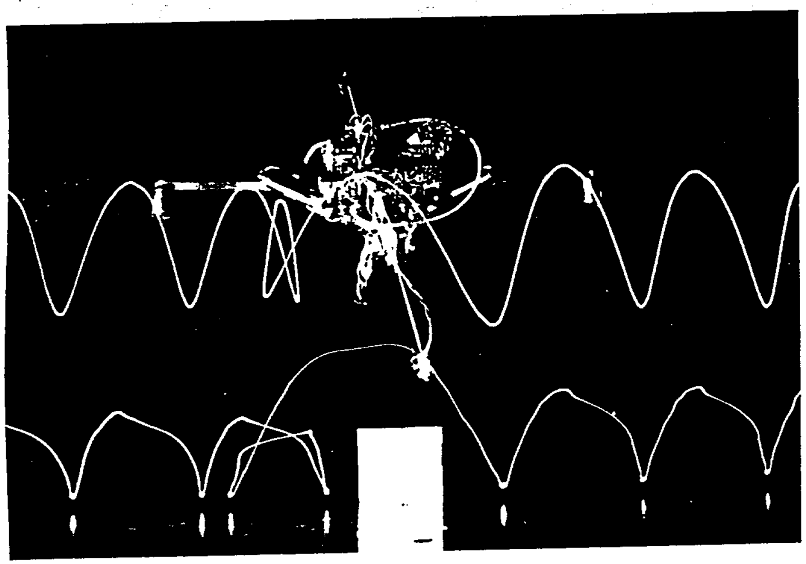


Figure 2.15. Hopping machine leaping over an obstacle. The machine approaches from right and continues to the left after the leap. Lights indicate the paths of the foot and hip. The sequence of operations used to make the leap is described in text. The obstacle is a stack of styrofoam blocks, 0.19 m tall and 0.15 m wide.

Improvements and Limitations

Each part of the control system described in this chapter applies a simple algorithm to a part of the overall locomotion task. None of the particular algorithms is sophisticated, and none were tuned for high performance. In fact, the purpose has been to focus on the shape of the problem and to identify an overall framework within which algorithms with well-defined goals can operate. With a framework and a working control system in place, the task of refining and optimizing the details of the individual components should be straightforward. None of our work has yet focused on this sort of tuning, but the following suggests a few things that are at the top of the list.

The algorithm that controls body attitude produces an asymmetrical oscillation in body attitude, as shown in figure 2.13. The average body angle deviates from zero in rough proportion to running speed, with a sudden uprighting of the body during the first part of stance. Ben Brown has suggested that it should be possible to reduce the asymmetry in these

body oscillations, eliminate the large error at touchdown, and in general, reduce the amount of work required of the servo that controls body attitude. This could be accomplished by designing the control system to permit the body to pitch back and forth in counterrotation with the leg. The idea is to control the average orientation of the body and leg, rather than the body attitude itself. The behavior would be described by

$$J\phi + J_\ell\theta = 0, \tag{2.8}$$

$$J\dot{\phi} + J_\ell\dot{\theta} = 0, \tag{2.9}$$

where J and J_ℓ are the moments of inertia of the body and leg about the hip. For steady-state undisturbed behavior, the control system would not have to exert any torque on the body. The passive pitching behavior would be the same as the nominal pitching behavior.

The approximations used by the forward speed control, both to estimate the length of the CG-print and to generate accelerations, are somewhat crude. The CG-print estimate works fine for a stiff leg and for low forward running speeds because both keep the leg nearly vertical all the time, but the estimate deteriorates when there are large excursions in leg angle. In this latter case, the axial leg force first slows down the body during the first part of stance and then accelerates it, resulting in an average forward speed that is less than the speed during flight and in a CG-print that is shorter than expected. The effect is a steady-state error in running speed that increases with both increasing forward speed and decreasing leg stiffness. Stentz (1983) proposed a more accurate prediction that should reduce the error in predicting the length of the CG-print. It takes into account the vertical speed of the body at touchdown and a model of the leg. The functional relationship between the desired net acceleration and the displacement of the foot, (2.3), is another prime candidate for improvement.

Another example of an optimization that could improve performance is what I call *ground speed matching*. When running at high speed, the foot should not merely be left motionless during touchdown but should accelerate backward with respect to the hip before contact until it is stationary in space. This matches the foot's backward speed to the ground's backward speed before touchdown. At lift-off the foot should continue moving backward until it is fully unloaded. Running animals match their feet to ground speed in this way, but the hopping machine does not. Matching the foot and ground speed at touchdown requires accurate synchronization of the foot's backward acceleration with the precise moment of touchdown.

A difficult problem in locomotion is to measure the external state of the system, such as the position, speed, and orientation of the body. The problem is that there is no permanent place to attach the sensors. For the implementation reported in this chapter, the tether mechanism was instrumented to provide this information. The pitch angle is measured by an electro-optical sensor mounted on the pivoting base of the tether, and the forward position of the body is measured by a potentiometer mounted on the pivot. In a sense, we cheated, because a real vehicle must perform these sensing functions with onboard instrumentation and the tether is not onboard. On the other hand, the planar hopping machine is not a prototype vehicle, but an apparatus for experiments. In any case, in the next chapter I described techniques that solve some of these external sensing problems in a more satisfying way.

Unlike natural legs that fold, the one-legged machine described in this section and the machines described later on in this book all use legs that telescope to change length. Is this important? In terms of the geometry needed to place the foot on a foothold, both sorts of legs have similar capabilities. Both telescoping and folding legs can also be designed to deliver equivalent forces to the body and the ground. The difference comes when considering the dynamics of the leg motion itself. For the intermediate-level view of locomotion we are considering here, these details are not too important. But when one begins to optimize the leg motion, these details will be important. An example is Mochon and McMahon's (1980) study of human leg motion during walking. They found that the leg acts like a compound pendulum that swings freely to move forward. When modeling this level of detail for a particular legged system and when concerned with performance and efficiency, a telescoping leg will not do. However, we find that telescoping legs capture a large part of what is important in legged locomotion while avoiding some of the complication—they are easier to model and to build.

The three-part control system emphasizes the separate actions of controlling the vertical bouncing motion, forward travel, and the attitude of the body. Although there are interactions between these activities, we have found that the dynamics are sufficiently separate to permit the control system to treat them separately—each part of the control system behaves as though it affects only the one variable it is supposed to control, and interactions show up as disturbances. This independence results in a particularly simple control design that is effective when the machine hops in place, translates from one point to another, accelerates to change running speed, and leaps.

An important characteristic of the control system for locomotion that we implemented is its once-per-hop method of operation. The controls for hopping and forward speed take action just once during each hopping cycle, ignoring the servos that work at the joint level. For instance, the control system positions the foot with respect to the center of mass at touchdown. If the forward speed is in error during a step, no action can be taken to correct it until the next step, when the foot lands on the ground again. The step becomes the basic unit of control. A similar description applies to the delivery of leg thrust that drives the hopping motion. This approach to control requires that the control system incorporate knowledge about the intrinsic mechanical behavior of the machine, so that acceptable behavior occurs within each cycle, between control actions. The tipping of an inverted pendulum and the bouncing of a ball represent this sort of knowledge.

The primary reason for using a one-legged apparatus was not to lay the groundwork for a one-legged vehicle. It was to focus on the general problem of active balance in dynamic legged locomotion in a way that could later be generalized for multilegged systems. If we ignore the third dimension, generalizing from the one-legged machine to the kangaroo hopping on two legs is straightforward. A direct comparison can be made between the motions of the hopping machine's one leg and kangaroo's pair of legs. The primary difference is that the kangaroo uses its tail to help compensate for the large sweeping motions of the legs so that the body need not react by pitching so much on each hop. A control system for a kangaroo might still regulate hopping height, body attitude, and velocity as before.

The generalization to multilegged systems that do not hop is also not too hard to imagine. Many characteristics of the running biped are similar to the running of a one-legged machine, including the alternation between stance and flight, the regular vertical oscillations, and the support provided by one leg at a time. In the case of the biped, the two legs always sweep in opposite directions, making rotations of the body unnecessary even without a tail. Think of a biped as a hopping machine that substitutes a different leg on each stride. The three-part decomposition can be employed as before. For a limited set of gaits, this approach can also be used to control running in the quadruped. The details of adapting these one-leg techniques and algorithms to multilegged systems are the topic of chapter 4.

Summary

In this chapter I described a machine that uses a particularly simple form of running: hopping on one leg. Study of this machine was motivated by three points: the importance of balance, the requirement that the legs be springy, and the difficulty of leg coordination. We found that control of the one-legged hopping machine can be decomposed into three separate parts. One part controls hopping height by delivering a fixed leg thrust during each hopping cycle. A second part of the control system regulates the forward rate of travel by placing the foot a specified distance in front of the hip as the machine approaches the ground on each step. The third part of the control system corrects the attitude of the body by servoing the hip during stance. A state machine provides the glue that synchronizes the control actions to the ongoing hopping behavior. The control system that results from the decomposition is simple:

Hopping:

Thrust for specified duration during stance.

Exhaust to specified pressure during flight.

Forward Speed:

Choose foot position $x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d).$

Convert to hip angle $\gamma_d = \phi - \arcsin\left(\frac{x_f}{r}\right).$

Servo hip angle $\tau = -k_p(\gamma - \gamma_d) - k_v(\dot{\gamma}).$

Body Attitude:

Servo body angle $\tau = -k_p(\phi - \phi_d) - k_v(\dot{\phi}).$

Experiments show that these algorithms provide good control of the machine. They maintain consistent hopping height, reaching equilibrium after a change within a few hopping cycles. The machine can run at speeds of up to 1.2 m/s, with speed regulation to about ±0.25 m/s, and can travel from place to place. A modification to the hopping control enabled the machine to leap over small obstacles.

Chapter 3

Hopping in Three Dimensions

At first glance, the running of an animal, say a horse, a human, or a kangaroo, appears to be a planar activity. The legs swing fore and aft while the body bobs up and down. Depending on the gait and the animal, the body may also pitch back and forth. Motions of the legs propel the animal forward and upward so the feet can recover to new footholds further along the path of travel, and they allow the animal to balance itself so that it does not tip over. Despite the appearance of planarity, however, animal locomotion takes place in three dimensions, where motions occur with six degrees of freedom.

The appearance of planarity in animal running led us to wonder if the techniques used for locomotion in the plane could be extended for locomotion in three dimensions. The dynamics for straight line running might be largely determined by motion occurring in the sagittal plane, with negligible influence from motion normal to the plane. If so then control systems for locomotion in three dimensions could avoid the complexity of three-dimensional dynamics.

To consider this question, we built a second one-legged hopping machine that operates without external support. It balances itself as it travels freely about the laboratory. It can hop in place, travel from point to point under velocity or position control, and maintain its balance when pushed (figure 3.1). The techniques used to control this machine are direct extensions of those described in the previous chapter for planar hopping, with surprisingly little extra complication. In particular, they preserve the decomposition of running into vertical hopping, forward travel, and posture. Control of this three-dimensional machine is the topic of this chapter.