

Haken-Kelso-Bunz model

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The Haken-Kelso-Bunz (HKB) Model was originally formulated in 1985 to account for some novel experimental observations on human bimanual coordination that revealed fundamental features of self-organization: multistability, phase transitions (switching) and hysteresis, a primitive form of memory. Self-organization refers to the spontaneous formation of patterns and pattern change in a nonequilibrium system composed of very many components that is open to the exchange of matter, energy and information with its surroundings. HKB uses the concepts of synergetics (order parameters, control parameters, instability, etc.) and the mathematical tools of nonlinearly coupled (nonlinear) dynamical systems to account for self-organized behavior both at the cooperative, coordinative level and at the level of the individual coordinating elements. The HKB model stands as a building block upon which numerous extensions and elaborations have been constructed. In particular, it has been possible to derive it from a realistic model of the cortical sheet in which neural areas undergo a reorganization that is mediated by intra- and inter-cortical connections (Jirsa, Fuchs & Kelso, 1998; see also Fuchs, Jirsa & Kelso, 2000). HKB stands as one of the cornerstones of coordination dynamics, an empirically grounded theoretical framework that seeks to understand coordinated behavior in living things.

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Introduction

The behaviors of animals and people are functionally ordered spatiotemporal patterns that arise in a system of very many neural, muscular and metabolic components that operate on different time scales. The ordering is such that we are often able to classify it, like the gaits of a horse, for example, or the limited number of basic sounds (the so-called 'phonemes') that are common to all languages. Given the ubiquity of coordinated behavior in living things, one might have expected its lawful basis to have been uncovered many years ago. Certainly attempts were made in the classical works of scientists like C. S. Sherrington (1906), E. von Holst (1937), R.W. Sperry (1961) and N. Bernstein (1967). One drawback to progress has been the absence of a model system that affords the precise analysis of behavioral patterns and pattern change both in terms of experimental data and theoretical tools. The HKB-model (after Haken, Kelso and Bunz) was the outcome of an experimental program of

research that aimed to understand: 1) the formation of ordered states of coordination in human beings; 2) the multistability of these observed states; and 3) the conditions that give rise to switching among coordinative states (for review, see Kelso, 1995). Since its publication in 1985, the HKB model has been elaborated and extended in numerous ways and at several different levels of analysis. Indeed, HKB is probably the most extensively tested quantitative model in the field of human movement behavior (Fuchs & Jirsa, 2008). Because it was the first to establish that coordination in a complex biological system is an emergent, self-organized process and because it was able to *derive* emergent patterns of coordinated behavior from nonlinear interactions among the component subsystems, HKB stands as a basic foundation for understanding coordination in living things.

Phase transitions ('switches') in coordinated movement

The experimental window into the self-organization of behavior was a paradigm introduced by S. Kelso (1981; 1984). HKB is the theoretical model that explicitly accounted for Kelso's observations and in turn predicted additional aspects. First the basic empirical facts are described; then these observations are mapped onto an explicit model; then the model is derived from a level below, namely the interacting subsystems. Kelso's original experiments dealt with rhythmical finger and hand movements in human beings. Many studies in humans and monkeys up to that time studied single limb movements. The Kelso experiments required the coordination between the index fingers of both hands. This precise coordination of the hands requires the coordination within and between the hemispheres of the brain, later studied using high density EEG and MEG arrays to record cortical activity (such work will not be described here, but see Kelso, et al. (1992) for original MEG work; Wallenstein, Kelso & Bressler (1995) for EEG correlates, and Jirsa, Fuchs & Kelso (1998) for cortical modelling thereof). The kinematic characteristics of bimanual movements were monitored using infrared light-emitting diodes attached to the moving parts and were detected by an optoelectronic camera system. On occasion the electromyographic activity of the muscles was also recorded using fine-wire platinum electrodes (e.g. Kelso & Scholz, 1985), thereby allowing a detailed examination at both kinematic and neuromuscular levels. Subjects oscillated their fingers rhythmically in the transverse plane (i.e., abduction-adduction) in one of two patterns, in-phase or anti-phase. In the former pattern, homologous muscles contract simultaneously; in the latter, the muscles contract in an alternating fashion. Subjects may increase the speed at which they perform these movements or they follow a pacing metronome whose oscillation frequency was systematically increased from 1.25 Hz to 3.50 Hz in steps of .25Hz that lasted up to 10 sec. Subjects were instructed to produce one full cycle of movement with each finger for each beat of the metronome. The following features were observed:

- when the subject begins in the anti-phase mode and speed of movement is increased, a spontaneous switch to symmetrical, in-phase movement occurs;
- this transition happens swiftly at a certain critical frequency;
- after the switch has occurred and the movement rate is now decreased the subject remains in the symmetrical mode, i.e. she does not switch back;
- no such transitions occur if the subject begins with symmetrical, in-phase movements.

Thus, while humans are able to produce two patterns at low frequency values, only one--the symmetrical, in-phase mode remains stable as frequency is scaled beyond a critical value. Questions of practice and learning different patterns of behavior have been studied at both behavioral (e.g., Zanone & Kelso, 1992) and brain levels (e.g., Jantzen, et al., 2002) but would require another article and will not be addressed further here.

Theoretical modeling: mapping behavior onto dynamics

The goal is to account for all the observed patterns of behavior with as small a number of theoretical concepts as possible. In order to understand the observed patterns and pattern switching, the following questions must be addressed:

1. Given that very many things can be experimentally measured but not all are likely to be relevant, what are the essential coordination variables or order parameters and how can their dynamics be characterized? Order parameters are quantities that allow for a usually low-dimensional description of the dynamical behavior of a complex, high-dimensional system
2. What are the control parameters that move the system through its coordinative states?
3. How are the subsystems and their interactions to be described?
4. Given a concise model that captures key experimental features, what new observations does it predict?

In a first step, the relative phase or phase relation ϕ between the fingers appears to be a suitable coordination variable or order parameter. The reasons are

$$\phi$$

characterizes the observed patterns of behavior; ϕ changes abruptly at the transition and is only weakly dependent on parameters outside the transition; and ϕ has very simple dynamics in which the behavioral patterns may be characterized as attractors. Since the frequency of oscillation is followed closely in the experiments and does not appear to be dependent on the system (e.g. it has been demonstrated to be effective also in studies of coordination between two people, Schmidt, et al., 1990), frequency is the control parameter.

The dynamics of ϕ can thus be determined from a few basic postulates:

1. The observed patterns of behavior at $\phi = 0$ deg. and $\phi = \pm 180$ deg. are modelled as fixed point attractors. The dynamics are therefore assumed to be purely relaxational. This is a minimality strategy in which only observed attractor types appear in the model;
2. The model must produce the observed patterns of relative phasing behavior: bistability at low frequencies, monostable beyond a critical frequency;
3. Only point attractors of the relative phase dynamics should appear;
4. Due to the fact that relative phase is a cyclic variable--meaning that if a multiple of 2π is added or subtracted the system must remain unchanged--any equation of motion has to be written in terms of periodic functions, i.e. sines and cosines. Thus a first symmetry argument dictates that the system must be invariant under shifts in the relative phase by multiples of 2π . A general equation of motion with this property reads

$$\dot{\phi} = a_0 + \sum_{k=1}^{\infty} \{a_k \cos(k\phi) + b_k \sin(k\phi)\} \quad (1)$$

A second symmetry argument comes from the left-right symmetry of the bimanual system itself. Exchanging the left with the right finger and vice-versa does not change the observed phenomena. The model is thus symmetric

under the transformation $\phi \rightarrow -\phi$, i.e. if we replace $\phi \rightarrow -\phi$ the equation remains the same. The power of symmetries in science cannot be overstated: here they restrict the equation of motion to a certain class of functions and even assist in eliminating half of them (the constant a_0 and the cosines). Of course much further work shows that this symmetry is not perfect. Nature thrives on broken symmetry and coordinated movement is no exception. (Among the factors that have been experimentally demonstrated to break the symmetry of HKB are handedness, hemispheric asymmetry, attentional allocation, intention to stabilize a particular finger-metronome relationship and so forth. All of these may be considered perturbations of HKB and may be included in a fine tuning of the modeling procedure). The simplest possible equation of motion --the HKB model--that captures all the observed facts is

$$\dot{\phi} = -a \sin \phi - 2b \sin 2\phi$$

The minus signs in front of the coefficients and the factor 2 in front of the b make life easier because the relevant regions of the parameter space may now be given by a and b positive, and the factor 2 allows the potential $V(\phi)$ to be defined without fractions.

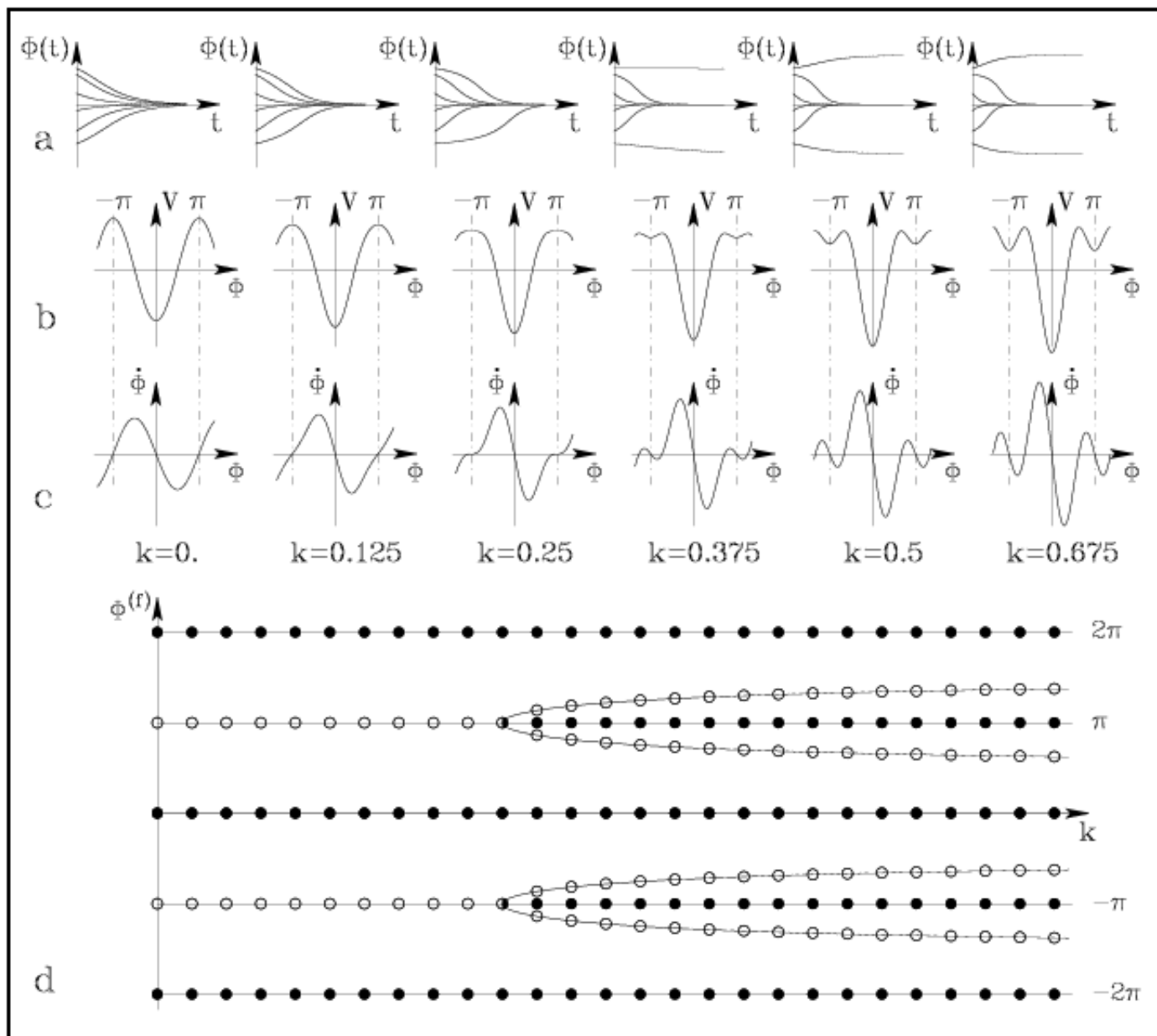
$$V(\phi) = -a \cos \phi - b \cos 2\phi$$

The equation of motion can be simplified further using rescaling, another powerful tool of nonlinear dynamical systems. Rescaling restricts the parameter space to a single positive parameter without changing any dynamical features

$$\dot{\phi} = -\sin \phi - 2k \sin 2\phi$$

The parameter k in the model (b/a in the original formulation) corresponds to the cycle to cycle period of the finger movements, that is, the inverse of the movement rate or oscillation frequency in the experiment. An increase in frequency thus corresponds to a decrease in k .

In order to determine whether this equation represents a valid theoretical model of the experimental findings one has to find the fixed points and check their stability. This means solving the equation for $\dot{\phi} = 0$. Haken, Kelso and Bunz (1985) showed that this equation captured all the observed experimental facts in the Kelso experiments.



The figure presents different ways to visualize the HKB model. Part (a) shows how the relative phase evolves in time from different initial conditions. Notice for high values of k corresponding to slow movements, initial conditions near in-phase and anti-phase converge to their respective attractors. Parts (b), (c) and (d) show the HKB potential (b), the phase portrait (c) and the bifurcation diagram, respectively. For $k > 0.25$ relative phase values of 0 and $\pm\pi$ are both stable, a condition called bistability. An increase in movement rate, starting in anti-phase, leads to a switch to in-phase at a critical frequency. Indeed, starting with a large k and decreasing k leads to a destabilization of the fixed point at π which becomes unstable at the value $k_c = 0.25$ and the system switches spontaneously into the in-phase pattern at $\phi = 0$. For parameter values smaller than 0.25 the fixed points at $\pm\pi$ are unstable and the only remaining one is stable at $\phi = 0$ corresponding to in-phase. Starting in the in-phase pattern for large k (slow movement) and decreasing k , does not lead to a transition because $\phi = 0$ is stable for all values of k . Likewise, beginning in the $\phi = 0$ pattern with a small k (fast movement) and slowing

the movement down does not cause behavior to change. Even beyond the critical value where anti-phase movement is possible, the system stays where it is. This is called hysteresis: there is no reason for the original pattern to change because the system is already in a stable coordinative state.

Theoretical Predictions and Experimental Confirmation

In HKB loss of stability, also called *dynamic instability*, causes switching to occur. One is free to inquire about the location of 'switches' inside the system but that is not the key to understanding what is going on. Stability can be measured in several ways:

1. **Critical slowing down.** If a small perturbation is applied to the system that drives it away from its stationary state, the time for the system to return to its stationary state (its local relaxation time) is a measure of its stability. The smaller the local relaxation time, the more stable the attractor. The less stable the pattern the longer it should take to return to the established pattern. HKB--or more correctly its stochastic equivalent (Schöner, Haken & Kelso, 1986)--predicts critical slowing down. That is, if the antiphase pattern is actually losing stability as the control parameter of frequency is increased, the local relaxation time should increase as the system approaches the critical point. Excellent agreement with theory was obtained in careful experiments (Scholz & Kelso, 1989; Scholz, Kelso & Schöner, 1987).
2. **Critical fluctuations.** A signature feature of non-equilibrium phase transitions in nature is the presence of critical fluctuations. If switching patterns of behavior is due to loss of stability, direct measures of fluctuations of the order parameter (relative phase) should be detectable as the critical point approaches. Experiments by Kelso et al. (1986) showed a striking enhancement of fluctuations (measured as the standard deviation of the continuous relative phase) for the antiphase pattern as the control parameter approached a critical value. No such increase was observed over the same parameter range for the in-phase pattern.

Deriving patterns of behavior and pattern change from subsystem interactions at a lower level

The HKB equation characterizes coordinated patterns of behavior and their pattern dynamics in terms of the order parameter or relative phase dynamics. However, it is important to recognize that the complete HKB model also *derives* these dynamics from a lower level. To accomplish this step one has to consider the subsystems and how these subsystems interact to produce coordinated states. This means it is necessary to provide a mathematical description of the fingers (or more generally the limbs) and a coupling between them. Again, it was very important to use experimental facts to guide theoretical modeling. Kinematic features of amplitude, frequency and velocity relationships were measured by Kelso, et al. (1981) and more rigorously by Kay, et al. (1987). In particular, the amplitude of individual finger oscillation was observed to decrease monotonically with frequency. Moreover, additional perturbation and phase resetting experiments by Kay, Kelso & Saltzman (1991) showed that individual hand movements returned to their cyclical trajectories with finite relaxation times. The HKB model thus maps the stable and reproducible oscillatory performance of each finger onto a limit cycle attractor in the x and \dot{x} phase plane. Again symmetry considerations play an important role. Finger movements consist of repetitive executions of flexion and extension in which one half cycle of flexion is approximately the inverse of one half cycle of extension. In other words, whether the finger is flexing or extending does not

essentially change the dynamics of the movement. For the equation of motion this means that if x and all its derivatives are substituted by $-x$ the equation must remain invariant. The equation of motion up to third order for the oscillation of a single limb takes the form

$$\ddot{x} + \epsilon\dot{x} + \omega^2 x + \gamma x^2 \dot{x} + \delta \dot{x}^3 = 0$$

This specific equation has been termed the "hybrid oscillator" because it consists of two types of oscillators known in the literature, i.e. the van der Pol oscillator for $\delta = 0$ and the Rayleigh oscillator for $\gamma = 0$. The reason to combine them is to get an accurate representation of the experimentally observed properties of single finger movements. Of course the main goal is to derive the HKB equation from the level of the individual components and their interaction. A crucial issue is the coupling function. In general, the coupling of two hybrid oscillators leads to a system of differential equations of the form

$$\ddot{x}_1 + \epsilon\dot{x}_1 + \omega_1^2 x_1 + \gamma x_1^2 \dot{x}_1 + \delta \dot{x}_1^3 = f_1 \{x_1, \dot{x}_1, x_2, \dot{x}_2\}$$

$$\ddot{x}_2 + \epsilon\dot{x}_2 + \omega_2^2 x_2 + \gamma x_2^2 \dot{x}_2 + \delta \dot{x}_2^3 = f_2 \{x_1, \dot{x}_1, x_2, \dot{x}_2\}$$

Notice that the same parameters ϵ , γ , and δ appear for both oscillators differing only in their eigenfrequencies ω_i (see Fuchs, et al., 1996). Haken, Kelso & Bunz (1985) considered a number of coupling structures for the observed phasing patterns and phase transitions. Linear couplings of position and its first order derivatives (velocity) are inadequate. Quadratic coupling terms violate symmetry requirements. Also, since the amplitudes of the oscillators are almost identical, a coupling based on the difference in the variables will act only as a small perturbation and not destroy the limit cycle structure of the oscillators. Hence, the simplest coupling that leads to the equation of motion for the relative phase is the sum of the linear term in the velocities and the cubic term in velocities times displacement squared

$$f_1 = \alpha(\dot{x}_1 - \dot{x}_2) + \beta(\dot{x}_1 - \dot{x}_2)(x_1 - x_2)^2 = (\dot{x}_1 - \dot{x}_2)\{\alpha + \beta(x_1 - x_2)^2\}$$

$$f_2 = \alpha(\dot{x}_2 - \dot{x}_1) + \beta(\dot{x}_2 - \dot{x}_1)(x_2 - x_1)^2 = (\dot{x}_2 - \dot{x}_1)\{\alpha + \beta(x_2 - x_1)^2\}$$

Using the above equations, Haken, Kelso & Bunz (1985) derived the final form of the dynamics of the order parameter relative phase as

$$\dot{\phi} = (\alpha + 2\beta r^2) \sin \phi - \beta r^2 \sin 2\phi$$

thereby establishing in a rigorous fashion the relation between the two levels of description.

The relation between parameters a and b at the collective, coordinative level, and the oscillator and coupling parameters r (amplitude), α and β

$$a = -(\alpha + 2\beta r^2) \quad \text{and} \quad b = \frac{1}{2} \beta r^2$$

yielded the critical frequency where the transition occurs is

$$k_c = \frac{b}{a} = \frac{1}{4} \quad \text{or with} \quad \frac{\frac{1}{2} \beta r^2}{-(\alpha + 2\beta r^2)} = \frac{1}{4}$$

which can be readily solved for the amplitude leading to

$$r_c^2 = -\frac{\alpha}{4\beta} .$$

Outlook

It is possible to provide only a hint of the various conceptual, methodological and practical developments that have arisen from the HKB model and the empirical observations that motivated it. These developments fall into several, by no means inclusive categories: a vast amount of research has been conducted based on the experimental paradigm itself and issues connected to the paradigm, including the roles of task context, biomechanical factors, perception, attention, cognitive demands, learning and memory (e.g. Carson, et al., 2000; Mechsner, et al., 2001; Pellecchia, Shockley & Turvey, 2005; Temprado, et al., 2002). Much of this research is a blend of both traditional and new methods and techniques. Issues of social coordination, the recruitment and coordination of multiple task components and the integration of movement with different sensory modalities have captured much recent interest. The latest noninvasive neuroimaging methods such as fMRI, MEG and high density EEG arrays are increasingly being used along with behavioral recording and analysis to identify the neural circuitry and mechanisms of pattern stability and switching (e.g., Aramaki, et al., 2005; Jantzen & Kelso, in press; Kelso, et al., 1998; Meyer-Lindenberg, et al., 2002; Swinnen, 2002). From a modeling point of view, major steps have included symmetry breaking of the HKB system (Kelso, et al., 1990) and its numerous conceptual consequences and paradigmatic applications, e.g. its role in the recruitment and coordination of multiple components; how it has revealed the balance of integrative and segregative processes in the brain (metastability). Discrete as well as rhythmic behaviors of individual and coupled systems have been studied (e.g. Schaal, et al., 2005) and accommodated in theoretical models (e.g., Jirsa & Kelso, 2005). HKB has also been extended to handle events at a neural level (Jirsa, Fuchs & Kelso, 1998). Although detailed anatomical architectures will always depend on specific contexts, the power of the approach is that it poses constraints on allowable types of architectures (see, e.g., Daffertshofer et al., 2005; Banerjee & Jirsa, 2006). When it comes to the brain, the need for at least a two-layer structure between functional units localized in the brain and the input and output components that are coordinated has been recognized by several research groups (e.g. Beek, Peper & Daffertshofer, 2002; Jirsa, Fuchs & Kelso, 1998). The incorporation of time delays into explicitly neural models, e.g. of interhemispheric coordination during bimanual and sensorimotor tasks is under active investigation, as are the behavioral, neural and modelling mechanisms underlying the different ways in which switching and elementary decision-making occur.

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See also

Coordination dynamics, Self-organization, Synchronization, Synergies

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